The Physics of Neutrinos

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Lectures :

1. Panorama of Experiments
2. Neutrino Oscillations
3. Models for Neutrino Masses
4. Neutrinos in Cosmology

Interlude:
SM & neutrinos

The Standard Model
(in a nutshell)

1961 - Glashow proposes $SU(2) \times U(1)$ as the local symmetry group for weak interactions

1964 - Salam and Ward use $SU(2) \times U(1)$ local to construct a model for electrons and muons

1967/8 - Weinberg and Salam, independently, propose a spontaneously broken $SU(2)_L \times U(1)_Y$ model for leptons. Quarks included in early 70's following Glashow, Iliopoulos and Maiani

1971 - 't Hooft proves renormalizability of SB gauge theories having interactions w/ operators of $d \leq 4$

1973 - Gross, Wilczek and Politzer showed that $SU(3)_c$ of strong interaction is asymptotically free

The Standard Model

$$SU(3)_C \times SU(2)_L \times U(1)_Y$$

Electroweak Symmetry Group

- **SU(2)$_L$**: weak isospin Group Generators: I_a ($a = 1,2,3$)

with $[I_a, I_b] = i\,\varepsilon_{abc}\, I_c$

e.g. in 2D representation $I_a = \tau_a/2$

- **U(1)$_Y$** : hypercharge Group Generator: Y

The action of Y on fermion fields is constrained by

Gell-Mann-Nishijima Relation

$$Q = I_3 + Y$$

The Standard Model

Representations of the fermion fields (which lead to the correct phenomenology) is

left-handed (L) chiral components: weak isospin doublets

LEPTONS

$$\mathbf{L_e} = \begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix}$$

$$\mathbf{L}_\mu = \begin{pmatrix} \nu_{\mu L} \\ \mu_L \end{pmatrix}$$

$$\mathbf{L}_\tau = \begin{pmatrix} \nu_{\tau L} \\ \tau_L \end{pmatrix}$$

$L_\alpha \equiv (2,-1/2)$

QUARKS

$$\mathbf{Q_u} = \begin{pmatrix} u_L \\ d_L \end{pmatrix}$$

$$\mathbf{Q_c} = \begin{pmatrix} c_L \\ s_L \end{pmatrix}$$

$$\mathbf{Q_t} = \begin{pmatrix} t_L \\ b_L \end{pmatrix}$$

$Q_\alpha \equiv (2,1/6)$

The Standard Model

Representations of the fermion fields (which lead to the correct phenomenology) is

right-handed (R) chiral components: weak isospin singlets

LEPTONS

e_R, μ_R, τ_R

$E_\alpha \equiv (1,-1)$

~~$\nu_{e_R}, \nu_{\mu_R}, \nu_{\tau_R}$~~

not present in the SM

QUARKS

u_R, c_R, t_R

$U_\alpha \equiv (1,4/6)$

$\alpha = u,c,t$

d_R, s_R, b_R

$D_\alpha \equiv (1,-2/6)$

$\alpha = d,s,b$

The Standard Model

Since L and R components of the fermion fields transform in different way, the presence of a bare mass term

$$\mathcal{L}_{\rm mass} \propto \bar{\rm f}{\rm f} = \bar{\rm f}_{\rm L}{\rm f}_{\rm R} + \bar{\rm f}_{\rm R}{\rm f}_{\rm L}$$

in the SM Lagrangian is forbidden by $SU(2)_L \times U(1)_Y$ symmetry ⇨ Fermion masses generated by the HIGGS MECHANISM

$SU(2)_L \times U(1)_Y \Rightarrow U(1)_Q$ after spontaneous symmetry breaking

EWSB

The Standard Model

fermion masses arise from Yukawa interactions

$$-\mathcal{L}_{\rm Y} = \mathbf{y}^{\mathbf{d}}_{\alpha\beta}\,\overline{\mathbf{Q}}_\alpha\,\mathbf{\Phi}\,\mathbf{D}_\beta + \mathbf{y}^{\mathbf{u}}_{\alpha\beta}\,\overline{\mathbf{Q}}_\alpha\,\tilde{\mathbf{\Phi}}\,\mathbf{U}_\beta + \mathbf{y}^{\ell}_{\alpha\beta}\overline{\mathbf{L}}_\alpha\,\mathbf{\Phi}\,\mathbf{E}_\beta + \text{h.c.}$$

$$\mathbf{\Phi}(\mathbf{x}) = \begin{pmatrix} \Phi^+(x) \\ \Phi^0(x) \end{pmatrix} \equiv (\mathbf{2}, \mathbf{1}/\mathbf{2}) \quad \tilde{\mathbf{\Phi}}(\mathbf{x}) = \mathbf{i}\tau_\mathbf{2}\mathbf{\Phi}(\mathbf{x})^* \equiv (\mathbf{2}, -\mathbf{1}/\mathbf{2})$$

$$\mathbf{\Phi} \rightarrow \langle\mathbf{\Phi}\rangle = \frac{\mathbf{1}}{\sqrt{\mathbf{2}}}\begin{pmatrix} 0 \\ \mathbf{v} \end{pmatrix} \qquad \mathbf{\Phi} = \frac{\mathbf{1}}{\sqrt{\mathbf{2}}}\begin{pmatrix} 0 \\ \mathbf{v}+\mathbf{h} \end{pmatrix}$$

Higgs acquires a vev

Unitary Gauge

since we do not ν_R have no mass @tree-level

but can they acquire mass by loop corrections ?

Can we have $m_\nu \neq 0$ in the SM ?

a loop correction could induce an effective mass term like ....

$$\frac{\mathbf{y}^{\nu}_{\alpha\beta}}{\mathbf{v}} \, \mathbf{\Phi} \, \mathbf{\Phi} \, \mathbf{L}_\alpha \, \mathbf{L}_\beta$$

But the SM has an accidental global symmetry

$$\mathbf{G}_{\rm SM} = \mathbf{U(1)_B} \times \mathbf{U(1)_{L_e}} \times \mathbf{U(1)_{L_\mu}} \times \mathbf{U(1)_{L_\tau}}$$

No ! Neutrinos have no mass in the SM!

Can we have $m_\nu \neq 0$ in the SM ?

a loop correction could induce an effective mass term like ....

$$\frac{\mathbf{y}^{\nu}_{\alpha\beta}}{\mathbf{v}}\,\mathbf{\Phi}\,\mathbf{\Phi}\,\mathbf{L}_{\alpha}\,\mathbf{L}_{\beta}$$

this term violates G_{SM}

But the SM has an accidental global symmetry

$$\mathbf{G}_{\mathrm{SM}} = \mathbf{U(1)_B} \times \mathbf{U(1)_{L_e}} \times \mathbf{U(1)_{L_\mu}} \times \mathbf{U(1)_{L_\tau}}$$

No ! Neutrinos have no mass in the SM!

Neutrino mass is Beyond SM Physics

(Next Lecture)

Ignore this for now ...

Lecture II

Neutrino Oscillations

"In as far as the neutrino masses are negligeable compared to the charged lepton masses, the observable effects of leptonic mixing angles are limited to fairly exotic effects such as neutrino oscillations."

(Froggatt and Nielsen, 1978

Neutrino Oscillations in Vacuum

First Ideas

1957 – B. Pontecorvo suggested $\nu \to \bar{\nu}$ oscillations in analogy to $K^0 \to \bar{K}^0$ ones

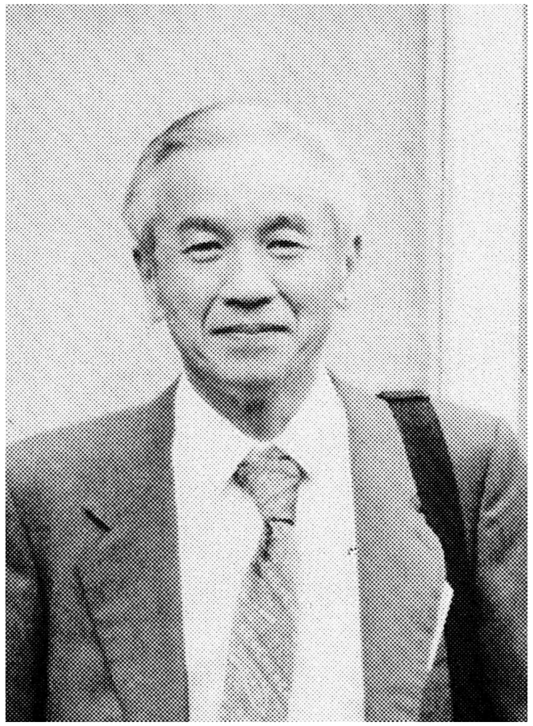
1962 – Flavor transitions $\nu_e \to \nu_\mu$ considered by Maki, Sakata and Nakagawa

Бруно Понтекорво

B. Pontecorvo

S. Sakata

Z. Maki



M. Nakagawa

Neutrino Masses & Mixings

$\nu_e, \nu_\mu, \nu_\tau \neq \nu_1, \nu_2, \nu_3$

(flavor or weak eigenstates) (mass eigenstates)

superposition of n light mass eigenstates

Note: if $|\bar{\nu}\rangle$: $U^* \to U$

$$|\nu_\alpha(x)\rangle = \sum_{i=1}^{n} U^*_{\alpha i} \int \frac{d^3p}{(2\pi)^3} f_j(\vec{p})\, e^{-iE_i(t-t_0)}\, e^{i\vec{p}(\vec{x}-\vec{x}_0)} |\nu_i\rangle$$

obs: indices L omitted !

neutrinos are produced by CC weak interactions as wave packets localized around a source position $x_0 = (t_0, \vec{x}_0)$

See E. Akhmedov and A. Smirnov, arXiv:0905.1903

We will derive the oscillation probability using plane waves - conceptually wrong but gives the right result much quicker

Neutrino Masses & Mixings

$\nu_e, \nu_\mu, \nu_\tau \neq \nu_1, \nu_2, \nu_3$

(flavor or weak eigenstates) (mass eigenstates)

mixing matrix

$$|\nu_\alpha\rangle = \sum_{\mathbf{i=1}}^{\mathbf{n}} \mathbf{U}^*_{\alpha\mathbf{i}} \, |\nu_\mathbf{i}\rangle \qquad \alpha = \mathbf{e}, \mu, \tau$$

state produced by CC interaction
superposition of n light mass eigenstates

$$|\nu_\alpha(\mathbf{t})\rangle = \sum_{\mathbf{i=1}}^{\mathbf{n}} \mathbf{U}^*_{\alpha\mathbf{i}} \, |\nu_\mathbf{i}(\mathbf{t})\rangle$$

after propagating for a time t
after traveling a distance $L \approx ct$

$$\mathbf{P}_{\alpha\beta}(\mathbf{t}) = |\mathcal{A}_{\alpha\beta}(\mathbf{t})|^2 = |\langle\nu_\beta|\nu_\alpha(\mathbf{t})\rangle|^2 = |\sum_{\mathbf{i=1}}^{\mathbf{n}}\sum_{\mathbf{j=1}}^{\mathbf{n}} \mathbf{U}^*_{\alpha\mathbf{i}}\mathbf{U}_{\beta\mathbf{j}}\langle\nu_\mathbf{j}(\mathbf{0})|\nu_\mathbf{i}(\mathbf{t})\rangle|^2$$

probability of detecting it as ν_β

Neutrino Oscillations in Vacuum

$$\mathbf{P}_{\alpha\beta}(\mathbf{t}) = |\mathcal{A}_{\alpha\beta}(\mathbf{t})|^2 = |\langle\nu_\beta|\nu_\alpha(\mathbf{t})\rangle|^2 = |\sum_{\mathbf{i}=1}^{\mathbf{n}}\sum_{\mathbf{j}=1}^{\mathbf{n}} \mathbf{U}^*_{\alpha\mathbf{i}}\mathbf{U}_{\beta\mathbf{j}}\langle\nu_\mathbf{j}(\mathbf{0})|\nu_\mathbf{i}(\mathbf{t})\rangle|^2$$

$$|\nu_\mathbf{i}(\mathbf{t})\rangle = \mathbf{e}^{-\mathbf{i}\frac{\mathbf{E_i}}{\hbar}\mathbf{t}}|\nu_\mathbf{i}(\mathbf{0})\rangle$$

all with the same momentum

[put back c and ħ for now]

$$\mathbf{P}_{\alpha\beta}(\mathbf{t}) = |\sum_{\mathbf{i}=1}^{\mathbf{n}} \mathbf{U}^*_{\alpha\mathbf{i}}\mathbf{U}_{\beta\mathbf{i}}\ \mathbf{e}^{-\mathbf{i}\frac{\mathbf{E_i}}{\hbar}\mathbf{t}}|^2$$

very relativistic neutrinos

$$\mathbf{E_i} = \sqrt{\mathbf{p^2c^2 + m_i^2c^4}} \approx \mathbf{p\,c} + \frac{\mathbf{m_i^2c^3}}{\mathbf{2\,p}} = \mathbf{E} + \frac{\mathbf{m_i^2c^4}}{\mathbf{2\,E}}$$

$$\mathbf{P}_{\alpha\beta}(\mathbf{L}) = \sum_{\mathbf{i,j}=1}^{\mathbf{n}} \mathbf{U}^*_{\alpha\mathbf{i}}\mathbf{U}_{\beta\mathbf{i}}\mathbf{U}_{\alpha\mathbf{j}}\mathbf{U}^*_{\beta\mathbf{j}}\ \mathbf{e}^{-\mathbf{i}\frac{\Delta\mathbf{m^2_{ij}c^3}}{\mathbf{2E}\hbar}\mathbf{L}}$$

to very good approximation

$$\Delta\mathbf{m^2_{ij}} = \mathbf{m_i^2} - \mathbf{m_j^2}$$

The Mixing Matrix U

$(n \times n)$ unitary matrix ⇨ n^2 real parameters

$n(n-1)/2$ mixing angles $\quad$ $n(n+1)/2$ phases

Dirac ν : $n + (n-1) = 2n-1$ phases can be absorbed in redefinitions of lepton fields

$n(n+1)/2 - (2n-1) = (n-1)(n-2)/2$ Dirac physical phases

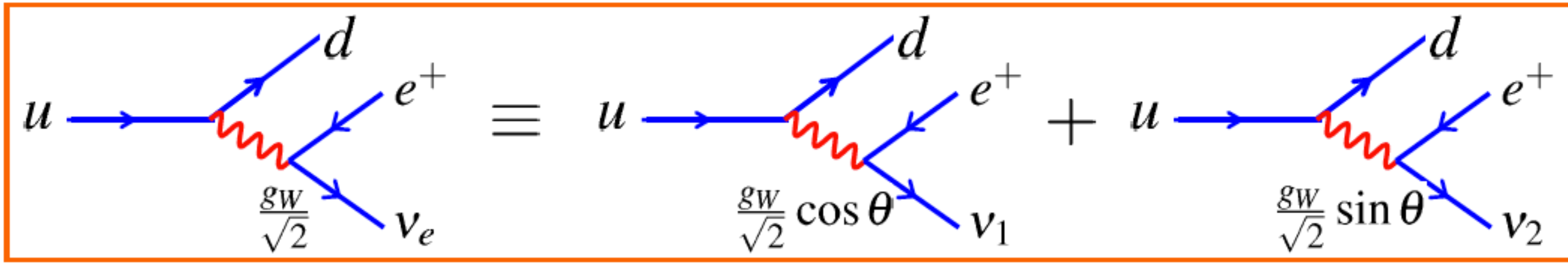
Majorana ν : only n phases can be absorbed in redefinitions of charged lepton fields

Dirac physical phases $+ (n-1)$ Majorana physical phases

do not enter oscillations

more on this next week ...

For Two Flavors



$$\mathbf{U} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

1 angle + 0 Dirac phases

oscillation probability

natural units ($c = \hbar = 1$)

survival probability

$$\mathbf{P_{ee} = 1 - P_{e\mu}}$$

$$\mathbf{P_{e\mu}(L) = \sin^2 2\theta \, \sin^2 \left(\boxed{\frac{\Delta m^2_{21} L}{4E}} \right)}$$

phase difference

equal masses → no oscillation

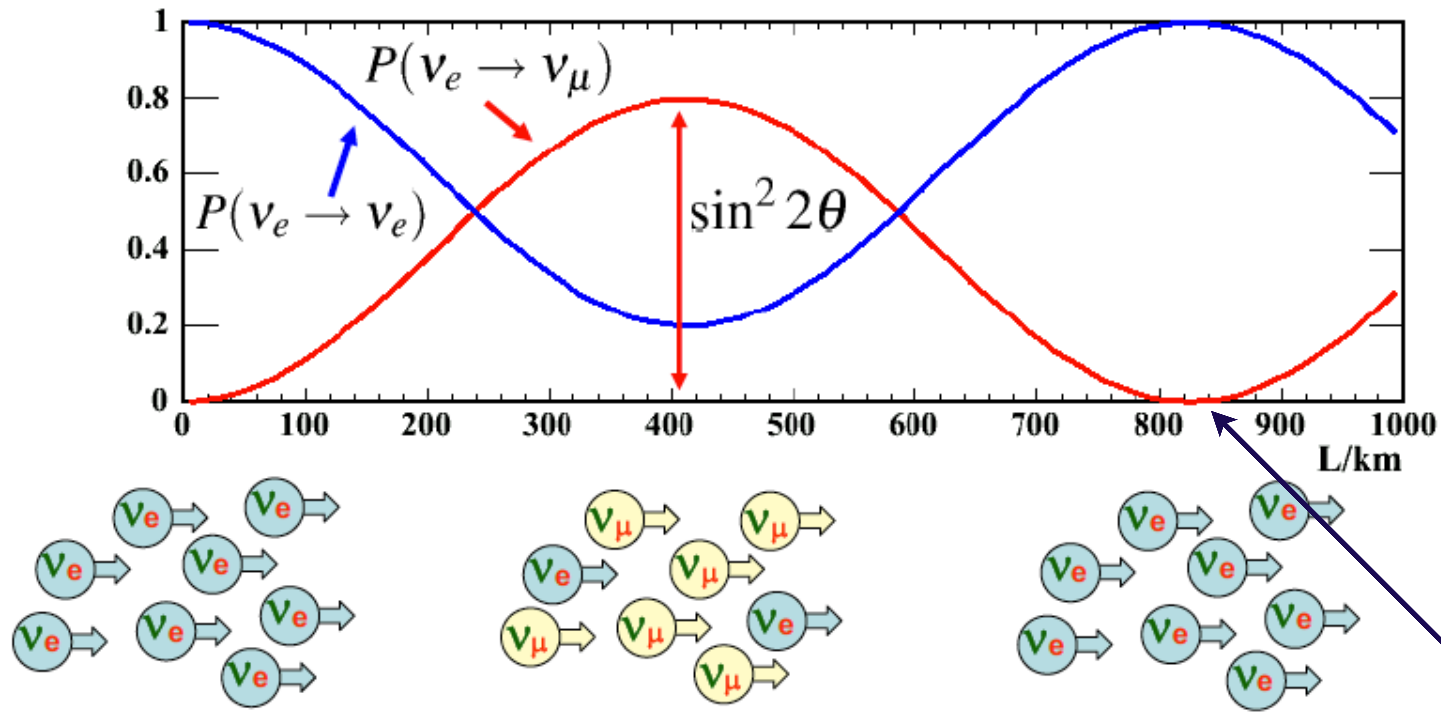
$$= \mathbf{\sin^2 2\theta \, \sin^2 \left(1.27 \frac{\Delta m^2_{21}}{eV^2} \frac{L}{m} \frac{MeV}{E} \right)}$$

L = baseline

For Two Flavors

oscillation length

•e.g. $\Delta m^2 = 0.003\,\text{eV}^2, \qquad \sin^2 2\theta = 0.8, \qquad E_\nu = 1\,\text{GeV}$



$$\mathbf{L_{osc}} = \frac{\mathbf{4\pi E}}{\mathbf{\Delta m^2_{21}}}$$

$$\sim \mathbf{2.5\,km}\,\frac{\mathbf{E\,(GeV)}}{\mathbf{\Delta m^2_{21}\,(eV^2)}}$$

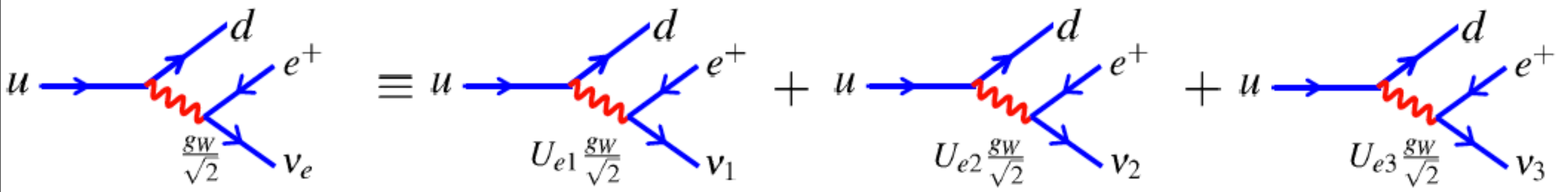
$$\mathbf{L_{osc}} \sim \mathbf{830\ km}$$

$$\mathbf{P_{e\mu}(L)} = \mathbf{\sin^2 2}\theta\ \sin^{\mathbf{2}}\left(\frac{\pi\,\mathbf{L}}{\mathbf{L_{osc}}}\right) \qquad \to \frac{\mathbf{1}}{\mathbf{2}}\sin^{\mathbf{2}}\mathbf{2}\theta$$

average regime

when can one observe oscillations?

For Three Flavors



$$\mathbf{U} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

standard parametrization - PMNS matrix

3 angles + 1 Dirac phase

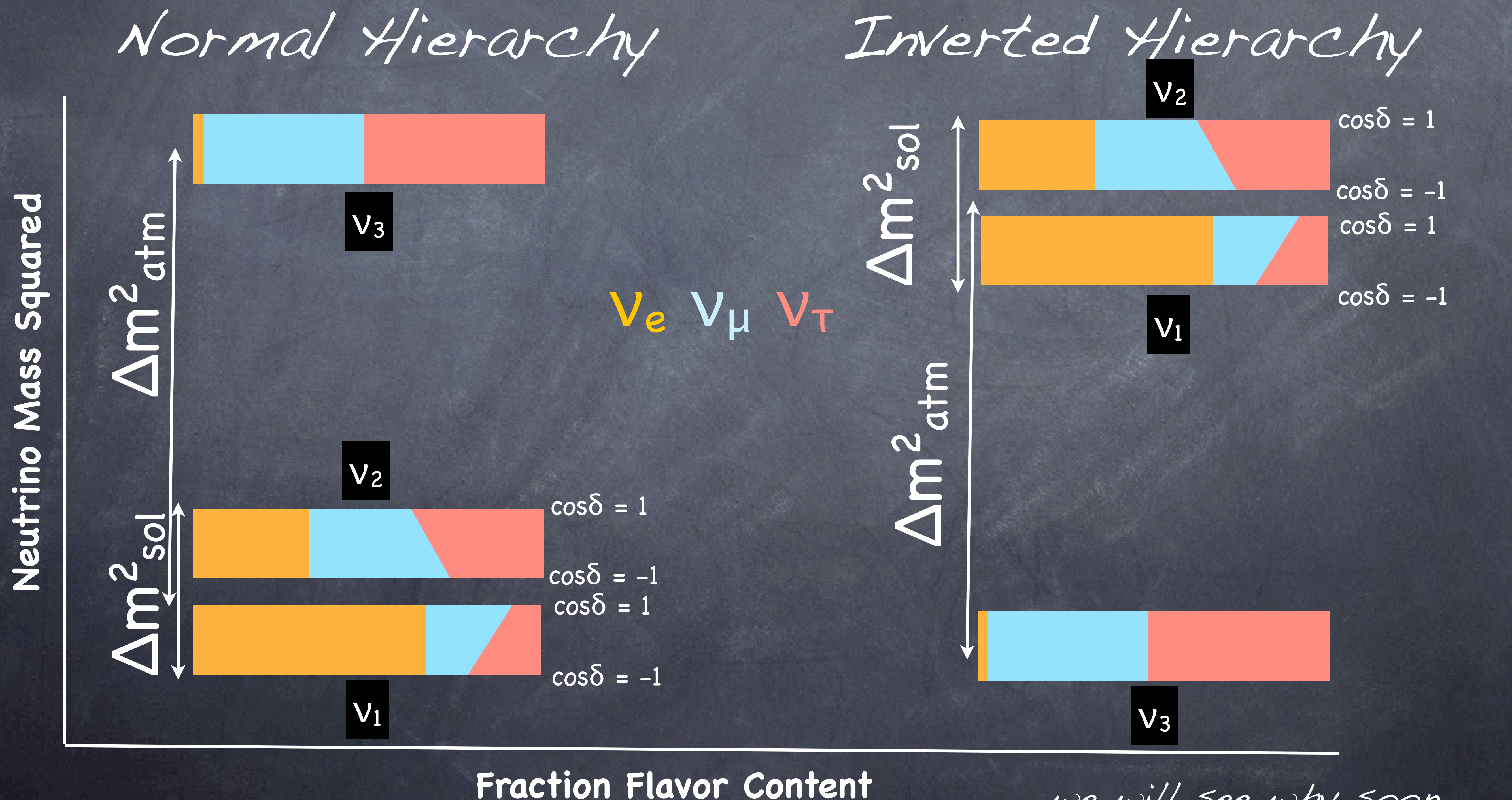
$$c_{ij} \equiv \cos\theta_{ij} \quad s_{ij} \equiv \sin\theta_{ij} \quad \theta_{ij} \in [0, \pi/2] \quad \delta \in [0, 2\pi]$$

$$\Delta m_{31}^2 = \Delta m_{32}^2 + \Delta m_{21}^2$$

if $\Delta m_{21}^2 \ll |\Delta m^2_{31}|$ and $s_{13} \to 0$ 12 and 23 sub-systems decouple

The Standard Framework

$\Delta m_{21}^2 = \Delta m^2_{sol} > 0 \quad \Delta m_{21}^2 << |\Delta m^2_{32}| \approx |\Delta m^2_{31}|$



we will see why soon

CPT in Neutrino Oscillations

CP symmetry

LH Particles ⇔ RH Antiparticles

RH Particles ⇔ LH Antiparticles

All Lorentz invariant, local Quantum Field Theories can be shown to be invariant under CPT (charge conjugation + parity + time reversal)

G. Lüders (1954), W. Pauli (1955), J.S.Bell (1954)

<u>No reason to think CPT is not conserved ...</u>

So if CP is conserved (violated) T is conserved (violated)

$CP : \nu_{\alpha L} \Leftrightarrow \overline{\nu}_{\alpha R} \Rightarrow U^*_{\alpha i} \Leftrightarrow U_{\alpha i} \; ; \quad \delta \Leftrightarrow -\delta$

$T : t \Leftrightarrow t_0 \Rightarrow \nu_{\alpha L} \Leftrightarrow \nu_{\beta L}$

~~CP~~ and ~~T~~ absent for 2 flavors . Its is a ≥ 3 flavor effect !

CPT in Neutrino Oscillations

Measures ~~CP~~:

$$\mathbf{\Delta P^{CP}_{\alpha\beta}} \equiv \mathbf{P}(\nu_\alpha \to \nu_\beta) - \mathbf{P}(\bar{\nu}_\alpha \to \bar{\nu}_\beta)$$

Measures ~~T~~:

$$\mathbf{\Delta P^{T}_{\alpha\beta}} \equiv \mathbf{P}(\nu_\alpha \to \nu_\beta) - \mathbf{P}(\nu_\beta \to \nu_\alpha)$$

Measures ~~CPT~~:

$$\mathbf{\Delta P^{CPT}_{\alpha\beta}} \equiv \mathbf{P}(\nu_\alpha \to \nu_\beta) - \mathbf{P}(\bar{\nu}_\beta \to \bar{\nu}_\alpha)$$

For 3 flavors

$$\mathbf{\Delta P^{CP}_{e\mu}} = \mathbf{\Delta P^{CP}_{\mu\tau}} = \mathbf{\Delta P^{CP}_{\tau e}} \equiv \mathbf{\Delta P}$$

$$\Delta P = -4\, s_{12}c_{12}s_{13}c_{13}^2 s_{23}c_{23}\, \sin\delta$$

Max. for $\delta = \pi/2$ or $3\pi/2$

$$\times \left[\sin\left(\frac{\Delta m^2_{12}}{2E}L\right) + \sin\left(\frac{\Delta m^2_{23}}{2E}L\right) + \sin\left(\frac{\Delta m^2_{31}}{2E}L\right)\right]$$

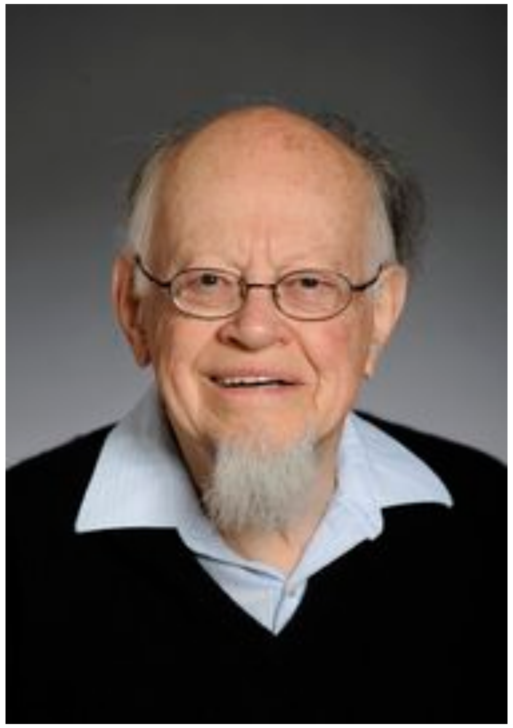
Very Difficult to measure ...

Neutrino Oscillations in Matter

The MSW Effect

1978 - Wolfenstein suggests matter can drastically impact neutrino oscillations

1985 - Mikheyev & Smirnov observe the possibility of resonance in flavor conversion



L. Wolfenstein

S. Mikheyev A. Yu Smirnov

How can matter affect neutrinos ?

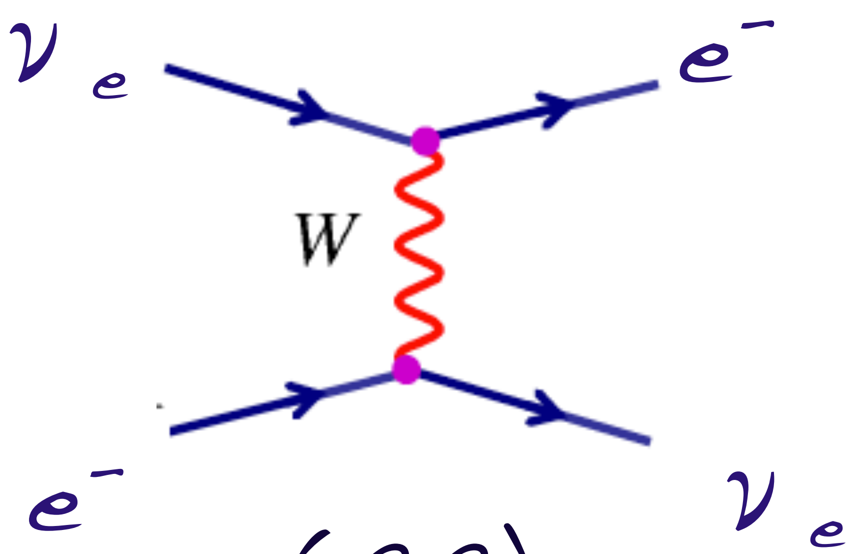
<u>incoherent process</u> (capture, finite angle scattering) $\sigma \propto G_F^2$

<u>coherent forward scattering</u>

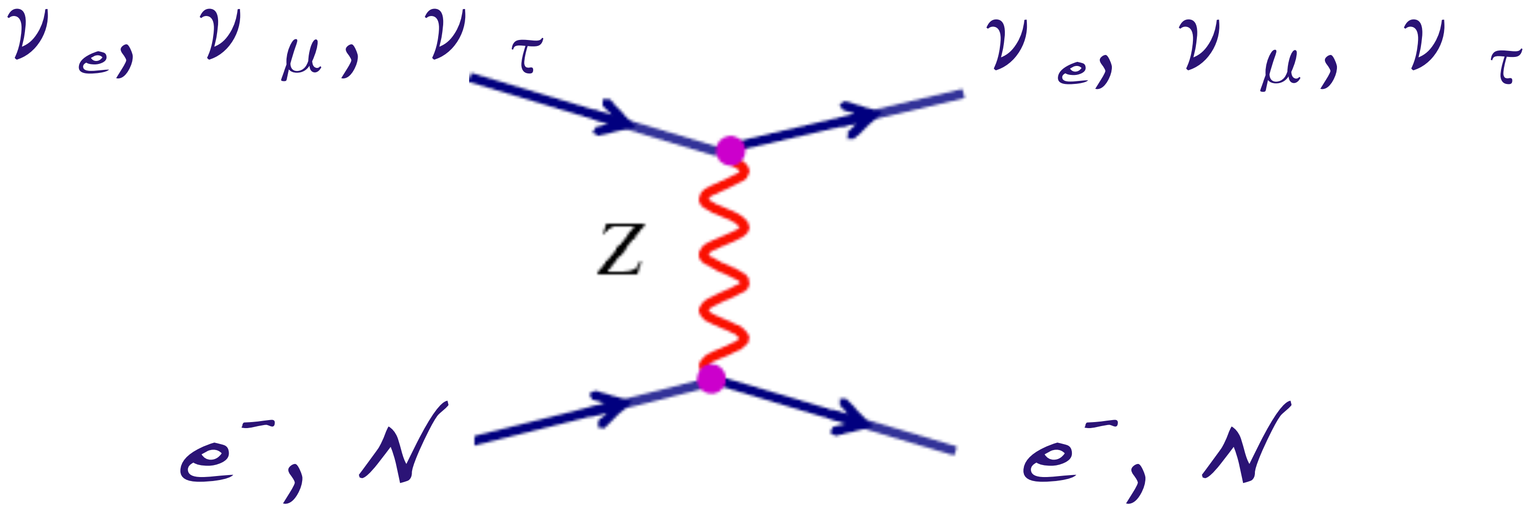
effects $\propto G_F$ a lot larger!

lead to effective potentials for ν's in matters $\propto G_F\, n$

Coherent Forward Scattering

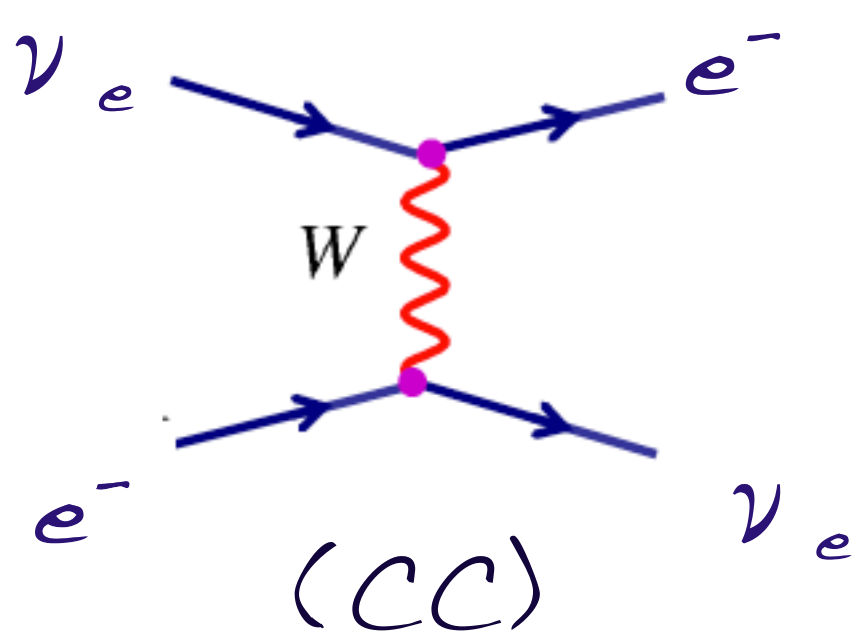


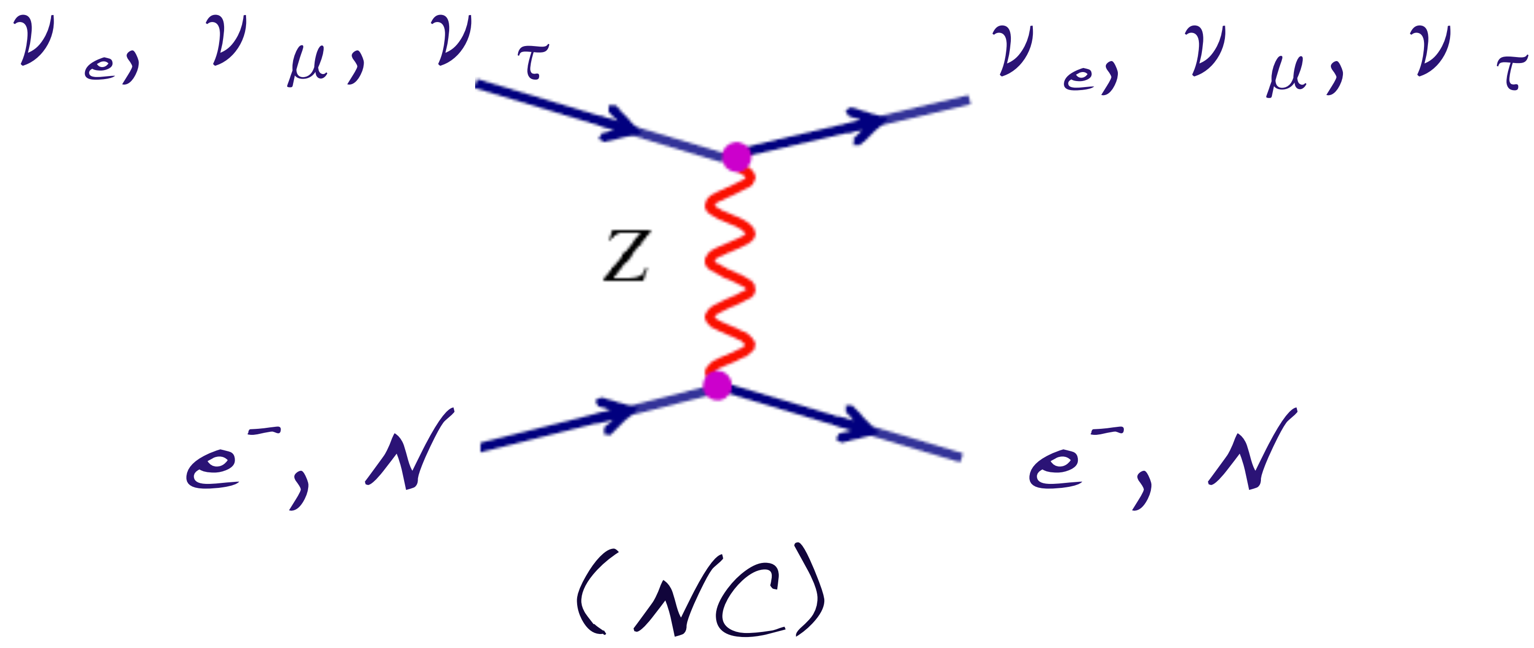
(CC)



(NC)

Coherent Forward Scattering





averaged over e- spin and summed over all e- in the medium

$$\mathbf{H_{CC}^{(e)}} = \sqrt{2}\,\mathbf{G_F} \int \mathbf{d^3p_e\, f(E_e, T)}$$
$$\times \langle< \mathbf{e(s, p_e)}|\mathbf{\bar{e}(x)}\gamma^\mu \mathbf{P_L}\, \nu_\mathbf{e}(\mathbf{x})\bar{\nu}_\mathbf{e}(\mathbf{x})\gamma_\mu \mathbf{P_L e(x)}|\mathbf{e(s, p_e)} >\rangle$$

$$= \sqrt{2}\mathbf{G_F}\bar{\nu}_\mathbf{e}(\mathbf{x})\gamma_\mu \mathbf{P_L}\nu_\mathbf{e}(\mathbf{x}) \int \mathbf{d^3p_e f(E_e, T)}\langle< \mathbf{e(s, p_e)}|\mathbf{\bar{e}(x)}\gamma^\mu \mathbf{P_L e(x)}|\mathbf{e(s, p_e)} >\rangle$$

f (Ee,T) homogeneous, isotropic and normalized

coherence → same e(s, pe) @ start and end

Coherent Forward Scattering

$$< \mathbf{e(s,p_e)}|\bar{\mathbf{e}}(\mathbf{x})\gamma^\mu \mathbf{P_L e(x)}|\mathbf{e(s,p_e)} >\rangle =$$

$$\frac{\mathbf{1}}{\mathbf{V}} < \mathbf{e(s,p_e)}|\bar{\mathbf{u}}_\mathbf{s}(\mathbf{p_e})\mathbf{a}^\dagger_\mathbf{s}(\mathbf{p_e})\gamma^\mu \mathbf{P_L a_s(p_e) u_s(p_e)}|\mathbf{e(s,p_e)} >$$

$$\langle \ldots \rangle = \mathbf{n_e(p_e)} \frac{\mathbf{1}}{\mathbf{2}} \sum_\mathbf{s} = \mathbf{n_e(p_e)} \frac{\mathbf{p_e^\mu}}{\mathbf{E_e}}$$

n_e = electron # density

isotropic medium

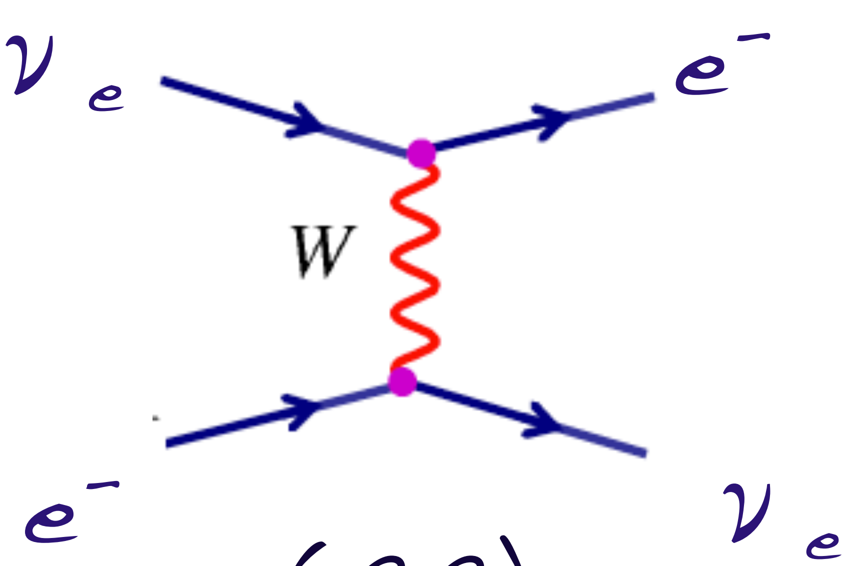
$$\int d^3 p_e \vec{p}_e f(E_e, T) = 0$$

$$\mathbf{H_c^{(e)}} = \sqrt{\mathbf{2}}\,\mathbf{G_F}\,\mathbf{n_e}\,\bar{\nu}_\mathbf{e}(\mathbf{x})\gamma_\mathbf{0}\mathbf{P_L}\nu_\mathbf{e}(\mathbf{x})$$

$$\mathbf{V_c} = \langle \nu_\mathbf{e}| \int \mathbf{d^3x}\,\mathbf{H_c^{(e)}} |\nu_\mathbf{e}\rangle = \sqrt{\mathbf{2}}\,\mathbf{G_F}\,\mathbf{n_e}$$

effective potential

Coherent Forward Scattering



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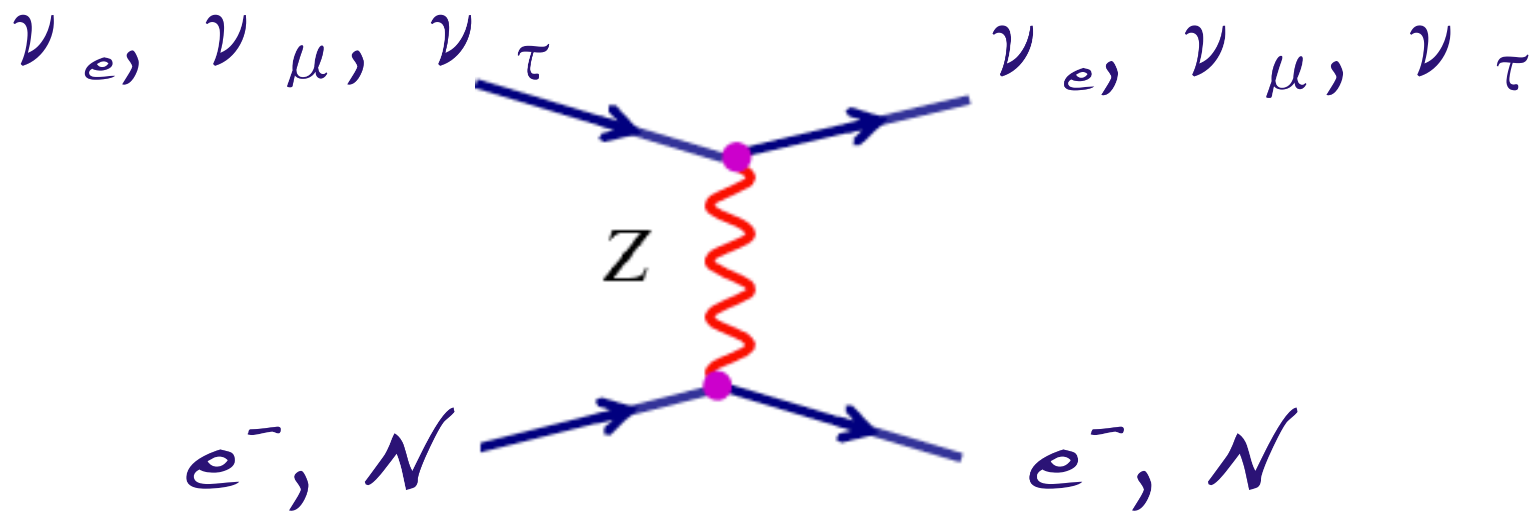


(NC)

$$V^{(e)}_{NC} = - V^{(p)}_{NC} \qquad V_{NC} = V^{(n)}_{NC}$$

$$V_e = V_C + V_{NC} \qquad V_\mu = V_\tau = V_{NC}$$

common phase

antineutrinos : $V \to -V$

ν Oscillations in Matter

Schrödinger picture

$$\mathbf{i}\frac{\mathbf{d}}{\mathbf{dt}}|\nu_\alpha(\mathbf{p},\mathbf{t})\rangle = \mathbf{H}|\nu_\alpha(\mathbf{p},\mathbf{t})\rangle \qquad |\nu_\alpha(\mathbf{p},\mathbf{0})\rangle \equiv |\nu_\alpha(\mathbf{p})\rangle$$

$$\mathcal{A}_{\alpha\beta}(\mathbf{p},\mathbf{t}) = \langle\nu_\beta(\mathbf{p})|\nu_\alpha(\mathbf{p},\mathbf{t})\rangle \qquad \mathcal{A}_{\alpha\beta}(\mathbf{p},\mathbf{0}) = \delta_{\alpha\beta}$$

flavor transition amplitude

$$\mathbf{i}\frac{\mathbf{d}}{\mathbf{dt}}\mathcal{A}_{\alpha\beta}(\mathbf{p},\mathbf{t}) = \langle\nu_\beta(\mathbf{p})|\mathbf{H}|\nu_\alpha(\mathbf{p},\mathbf{t})\rangle = \langle\nu_\beta(\mathbf{p})|\mathbf{H_0}|\nu_\alpha(\mathbf{p},\mathbf{t})\rangle + \langle\nu_\beta(\mathbf{p})|\mathbf{H_I}|\nu_\alpha(\mathbf{p},\mathbf{t})\rangle$$

$$\langle\nu_\beta(\mathbf{p})|\mathbf{H_0}|\nu_\alpha(\mathbf{p},\mathbf{t})\rangle = \sum_\rho \langle\nu_\beta(\mathbf{p})|\mathbf{H_0}|\nu_\rho(\mathbf{p})\rangle\langle\nu_\rho(\mathbf{p})|\nu_\alpha(\mathbf{p},\mathbf{t})\rangle$$

$$= \sum_\rho \sum_{\mathbf{j}} \mathbf{U}^*_{\rho\mathbf{j}}\,\mathbf{U}_{\beta\mathbf{j}}\,\mathbf{E_j}\,\mathcal{A}_{\alpha\rho}(\mathbf{p},\mathbf{t})$$

$$\langle\nu_\beta(\mathbf{p})|\mathbf{H_I}|\nu_\alpha(\mathbf{p},\mathbf{t})\rangle = \sum_\rho \underbrace{\langle\nu_\beta(\mathbf{p})|\mathbf{H_I}|\nu_\rho(\mathbf{p})\rangle}_{\delta_{\beta\rho}\mathbf{V}_\beta}\mathcal{A}_{\alpha\rho}(\mathbf{p},\mathbf{t})$$

ν Oscillations in Matter

$$\mathbf{i}\frac{\mathbf{d}}{\mathbf{dt}}\mathcal{A}_{\alpha\beta} = \sum_{\rho}\left(\sum_{\mathbf{j}} \mathbf{U}^*_{\rho\mathbf{j}}\,\mathbf{U}_{\beta\mathbf{j}}\,\mathbf{E}_{\mathbf{j}} + \delta_{\beta\rho}\mathbf{V}_{\beta}\right)\mathcal{A}_{\alpha\rho}(\mathbf{p},\mathbf{t})$$

ultra-relativistic ν : $E_j = E + m^2_j/(2E)$ $\quad t \approx r$

$V_e = V_c + V_{NC}$ $\qquad V_\mu = V_\tau = V_{NC}$

$$\mathbf{i}\frac{\mathbf{d}}{\mathbf{dr}}\mathcal{A}_{\alpha\beta} = (\mathbf{E}+\mathbf{V_{NC}})\mathcal{A}_{\alpha\beta}(\mathbf{p},\mathbf{r}) + \sum_{\rho}\left(\sum_{\mathbf{j}} \mathbf{U}_{\beta\mathbf{j}}\,\frac{\mathbf{m_j^2}}{\mathbf{2E}}\,\mathbf{U}^*_{\rho\mathbf{j}} + \delta_{\rho\mathbf{e}}\delta_{\beta\mathbf{e}}\mathbf{V_c}\right)\mathcal{A}_{\alpha\rho}(\mathbf{p},\mathbf{r})$$

$$\mathcal{A}'_{\alpha\beta}(\mathbf{p},\mathbf{r}) = \mathcal{A}_{\alpha\beta}(\mathbf{p},\mathbf{r})\,\mathbf{e}^{\mathbf{iE\,r}+\mathbf{i}\int_0^{\mathbf{r}}\mathbf{V_{NC}}(\mathbf{x}')\mathbf{dx}'}$$ global phase

ν Oscillations in Matter

$$\mathbf{i}\frac{\mathbf{d}}{\mathbf{dr}}\mathcal{A}_{\alpha\beta} = (\mathbf{E}+\mathbf{V_{NC}})\mathcal{A}_{\alpha\beta}(\mathbf{p},\mathbf{r}) + \sum_{\rho}\left(\sum_{\mathbf{j}}\mathbf{U}_{\beta\mathbf{j}}\,\frac{\mathbf{m_j^2}}{\mathbf{2E}}\,\mathbf{U}^*_{\rho\mathbf{j}} + \delta_{\rho\mathbf{e}}\delta_{\beta\mathbf{e}}\mathbf{V_c}\right)\mathcal{A}_{\alpha\rho}(\mathbf{p},\mathbf{r})$$

define

$$\mathcal{A}'_{\alpha\beta}(\mathbf{p},\mathbf{r}) = \mathcal{A}_{\alpha\beta}(\mathbf{p},\mathbf{r})\;\mathbf{e}^{\mathbf{iE\,r+i}\int_0^{\mathbf{r}}\mathbf{V_{NC}}(\mathbf{x}')\mathbf{dx}'}$$

global phase

$$\mathbf{i}\frac{\mathbf{d}}{\mathbf{dr}}\mathcal{A}'_{\alpha\beta}(\mathbf{p},\mathbf{r}) = \mathbf{e}^{\mathbf{iE\,r+i}\int_0^{\mathbf{r}}\mathbf{V_{NC}}(\mathbf{x}')\mathbf{dx}'}\left(-\mathbf{E}-\mathbf{V_{NC}}+\mathbf{i}\frac{\mathbf{d}}{\mathbf{dr}}\right)\mathcal{A}_{\alpha\beta}$$

$$\mathbf{i}\frac{\mathbf{d}}{\mathbf{dr}}\mathcal{A}'_{\alpha\beta} = \sum_{\rho}\left(\sum_{\mathbf{j}}\mathbf{U}_{\beta\mathbf{j}}\,\frac{\mathbf{m_j^2}}{\mathbf{2E}}\,\mathbf{U}^*_{\rho\mathbf{j}} + \delta_{\rho\mathbf{e}}\delta_{\beta\mathbf{e}}\mathbf{V_c}\right)\mathcal{A}'_{\alpha\rho}(\mathbf{p},\mathbf{r})$$

so $\mathbf{P}_{\alpha\beta} = |\mathcal{A}_{\alpha\beta}|^{\mathbf{2}} = |\mathcal{A}'_{\alpha\beta}|^{\mathbf{2}}$

The Standard Framework

$$\mathrm{i}\,\frac{\mathrm{d}}{\mathrm{dr}}\begin{pmatrix}\mathcal{A}_{\alpha e}\\ \mathcal{A}_{\alpha\mu}\\ \mathcal{A}_{\alpha\tau}\end{pmatrix} = \left(\frac{1}{2\mathrm{E}}\,\mathbf{U}\mathbf{M}^2\mathbf{U}^\dagger + \mathbf{A}\right)\begin{pmatrix}\mathcal{A}_{\alpha e}\\ \mathcal{A}_{\alpha\mu}\\ \mathcal{A}_{\alpha\tau}\end{pmatrix}$$

evolution of neutrino amplitudes

$$\mathbf{M} = \begin{pmatrix} 0 & 0 & 0\\ 0 & \Delta m_{21}^2 & 0\\ 0 & 0 & \Delta m_{31}^2 \end{pmatrix}$$

$$\mathbf{A} = \begin{pmatrix} \mathrm{V_e} & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0 \end{pmatrix}$$

$$\mathrm{V_e} = \sqrt{2}\,\mathrm{G_F n_e} \sim 7.6\,\mathrm{Y_e}\,\frac{\rho}{10^{14}\mathrm{g/cm^3}}\,\mathrm{eV}$$

$\rho \sim 10\,\mathrm{g/cm^3}\ \mathrm{V_e} \sim 10^{-13}\,\mathrm{eV}$ Earth's core

$\rho \sim 100\,\mathrm{g/cm^3}\ \mathrm{V_e} \sim 10^{-12}\,\mathrm{eV}$ Sun's core

Two Flavors in Matter

$$\mathbf{i}\,\frac{\mathbf{d}}{\mathbf{dr}}\begin{pmatrix}\mathcal{A}_{\alpha e}\\ \mathcal{A}_{\alpha\mu}\end{pmatrix} = \begin{pmatrix}-\frac{\Delta m^2_{21}}{4E}\cos 2\theta + V_e & \frac{\Delta m^2_{21}}{4E}\sin 2\theta\\ \frac{\Delta m^2_{21}}{4E}\sin 2\theta & \frac{\Delta m^2_{21}}{4E}\cos 2\theta\end{pmatrix}\begin{pmatrix}\mathcal{A}_{\alpha e}\\ \mathcal{A}_{\alpha\mu}\end{pmatrix}$$

constant matter density

$$\sin^{\mathbf{2}}\theta_{\mathbf{m}} = \frac{\mathbf{1}}{\mathbf{2}}\left[\mathbf{1} + \frac{(\mathbf{A} - \mathbf{\Delta m^2_{21}}\cos \mathbf{2}\theta)}{\mathbf{\Delta m^2_m}}\right]$$

$$\mathbf{\Delta m^2_m} = \sqrt{(\mathbf{\Delta m^2_{21}}\cos \mathbf{2}\theta - \mathbf{A})^{\mathbf{2}} + (\mathbf{\Delta m^2_{21}}\sin \mathbf{2}\theta)^{\mathbf{2}}}$$

will only happen for neutrinos if $\Delta m^2_{21} > 0$

$$\mathbf{A} \equiv \mathbf{2}\sqrt{\mathbf{2}}\,\mathbf{E}\,\mathbf{G_F}\,\mathbf{ne}$$

$$\sqrt{\mathbf{2}}\,\mathbf{G_F}\,\mathbf{ne} = \frac{\mathbf{\Delta m^2_{21}}}{\mathbf{2E}}\cos \mathbf{2}\theta$$

MSW resonance conditions

Two Flavors in Matter

$$\sin^2 \theta_m = \frac{1}{2}\left[1 + \frac{(A - \Delta m^2_{21} \cos 2\theta)}{\Delta m^2_m}\right]$$

MSW resonance conditions

$\theta_m = 45°$ maximal mixing

$$\sqrt{2}\, G_F\, n_e = \frac{\Delta m^2_{21}}{2E} \cos 2\theta$$

$n_e = n_e^{res}$

variable matter density

$$|\nu_e\rangle = \cos\theta_m |\nu_{1m}\rangle + \sin\theta_m |\nu_{2m}\rangle$$

interaction eigenstates

instantaneous eigenstates in matter

$$|\nu_\mu\rangle = -\sin\theta_m |\nu_{1m}\rangle + \cos\theta_m |\nu_{2m}\rangle$$

$n_e >> n_e^{res}$ $\theta_m = 90°$

$\nu_2 \rightarrow \nu_e$

$n_e << n_e^{res}$ $\theta_m \approx 0°$

Two Flavors in Matter

variable matter density

instantaneous eigenstates in matter

$$i\frac{d}{dr}\begin{pmatrix}\nu_{1m}\\ \nu_{2m}\end{pmatrix} = \frac{1}{4E}\begin{pmatrix}-\Delta m_m^2 & -4iE\,d\theta_m(r)/dr\\ 4iE\,d\theta_m(r)/dr & \Delta m_m^2\end{pmatrix}\begin{pmatrix}\nu_{1m}\\ \nu_{2m}\end{pmatrix}$$

adiabatic transitions:

if $\Delta m_m^2 >> 4E\,d\theta_m(r)/dr$

instantaneous mass eigenstates behave like energy eigenstates → they do not mix on evolution

$$P(\nu_e \to \nu_e) = \cos^2\theta_m \cos^2\theta + \sin^2\theta_m \sin^2\theta + \frac{1}{2}\sin 2\theta_m \sin 2\theta \cos\left(\frac{\delta(r)}{2E}\right)$$

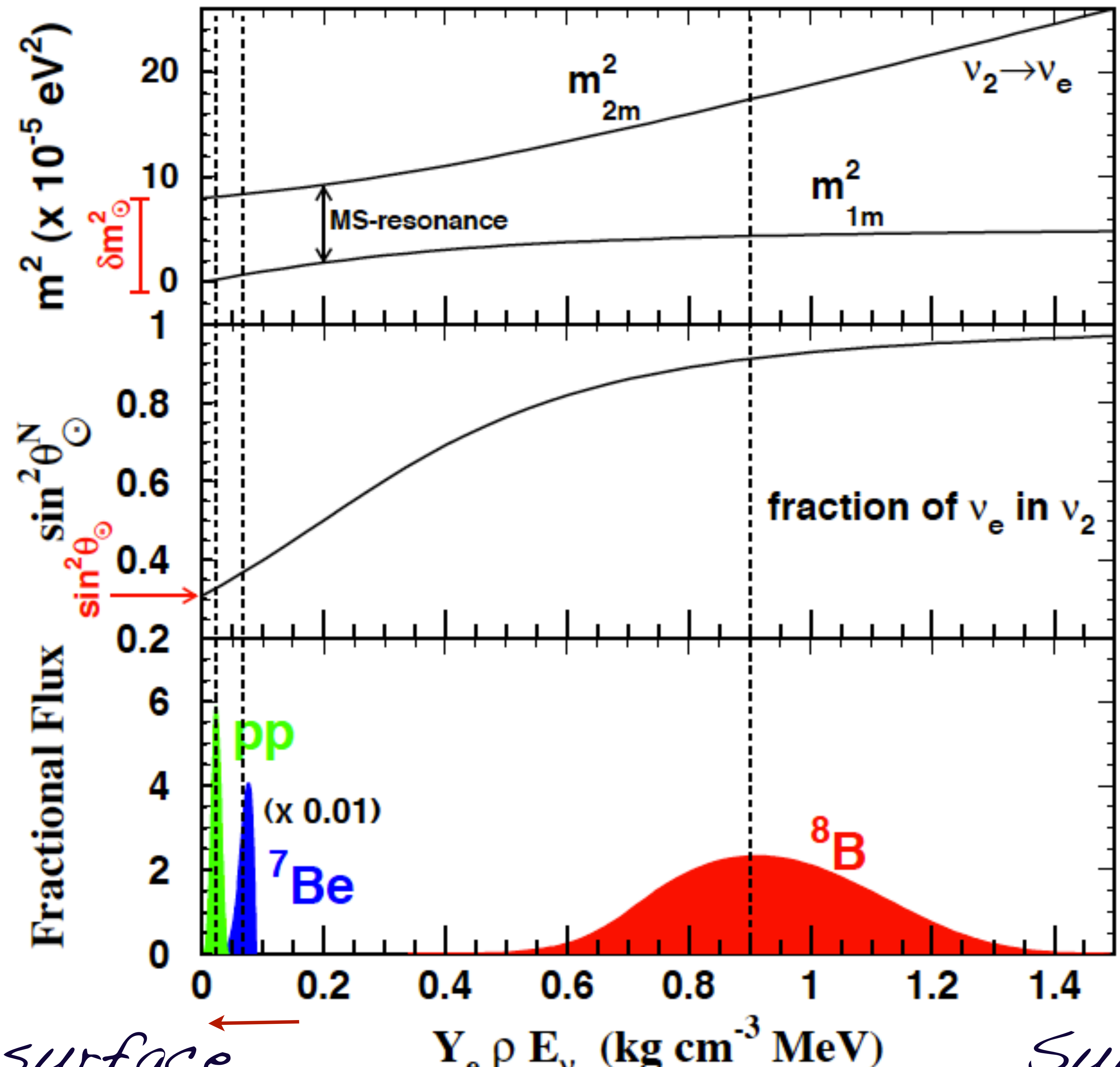
$$\delta(r) = \int_{r0}^{r} \Delta m_m^2(r')\,dr'$$

Sun : $\delta(r) >> E$

averaged

Two Flavors in the Sun

$$\mathbf{P}(\nu_\mathbf{e} \rightarrow \nu_\mathbf{e}) = \cos^\mathbf{2}\theta_\mathbf{m}\cos^\mathbf{2}\theta + \sin^\mathbf{2}\theta_\mathbf{m}\sin^\mathbf{2}\theta$$



$\sin^2\theta_m$

$\nu_e \equiv \nu_2$

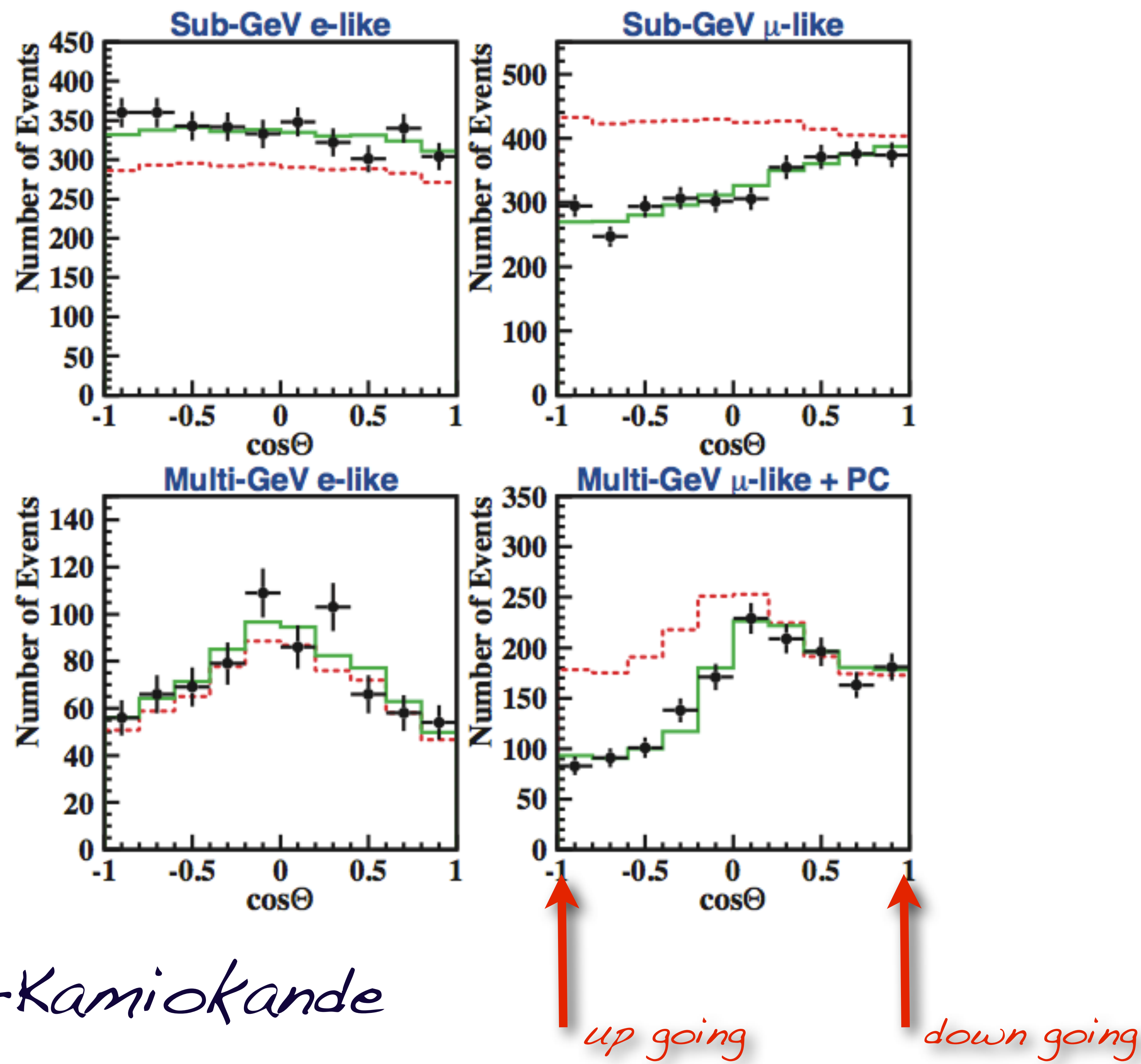
Sun's core

Sun's surface ←

→ Sun's core

Revisiting Experiments

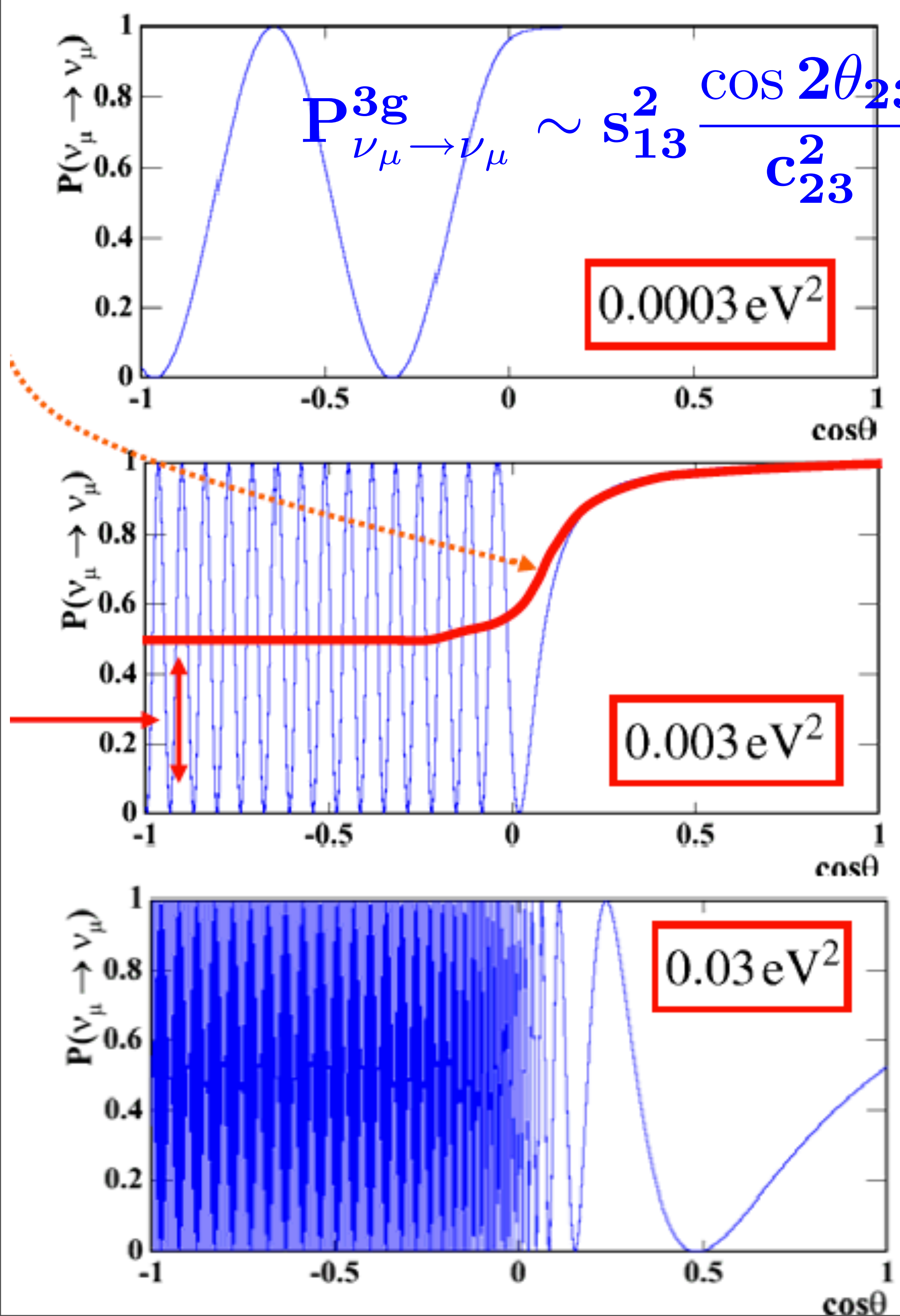
Atmospheric Neutrinos

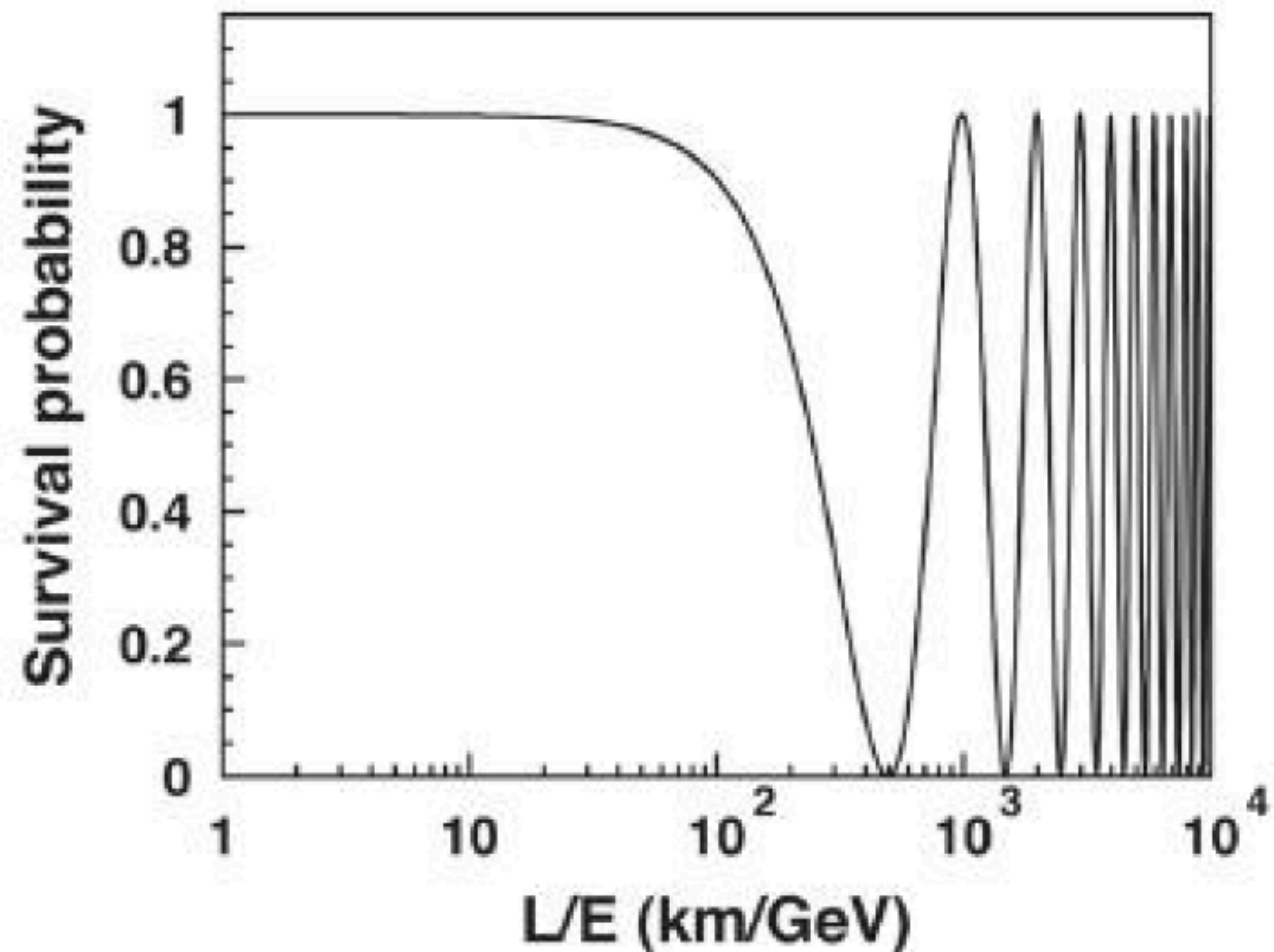


Super-Kamiokande

Atmospheric Neutrinos

$$P^{3g}_{\nu_\mu\to\nu_\mu} \sim s^2_{13}\frac{\cos 2\theta_{23}}{c^2_{23}} + \left(1 - s^2_{13}\frac{\cos 2\theta_{23}}{c^2_{23}}\right) P^{2g}_{\nu_\mu\to\nu_\mu}(\Delta m^2_{31}, \theta_{23})$$



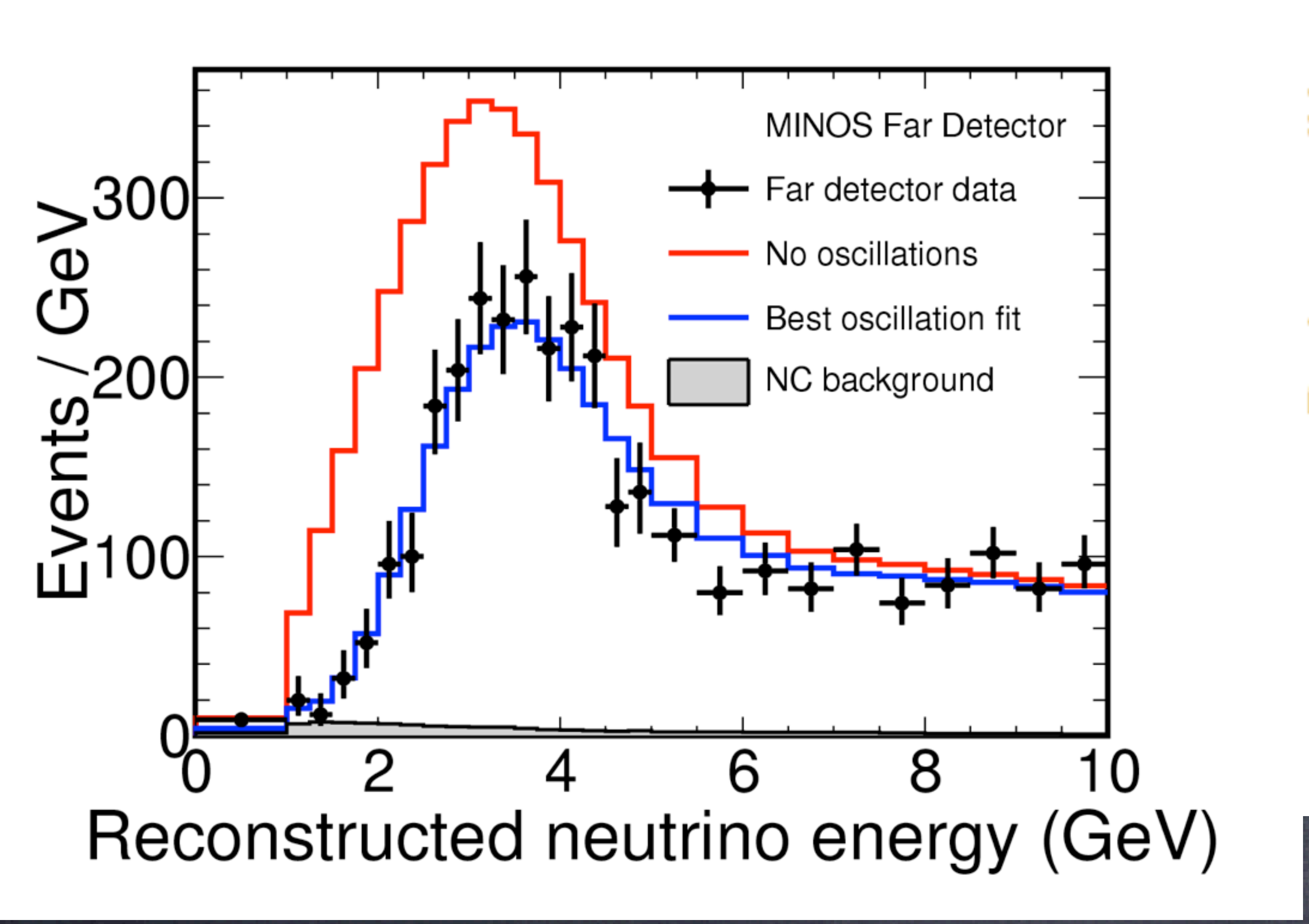


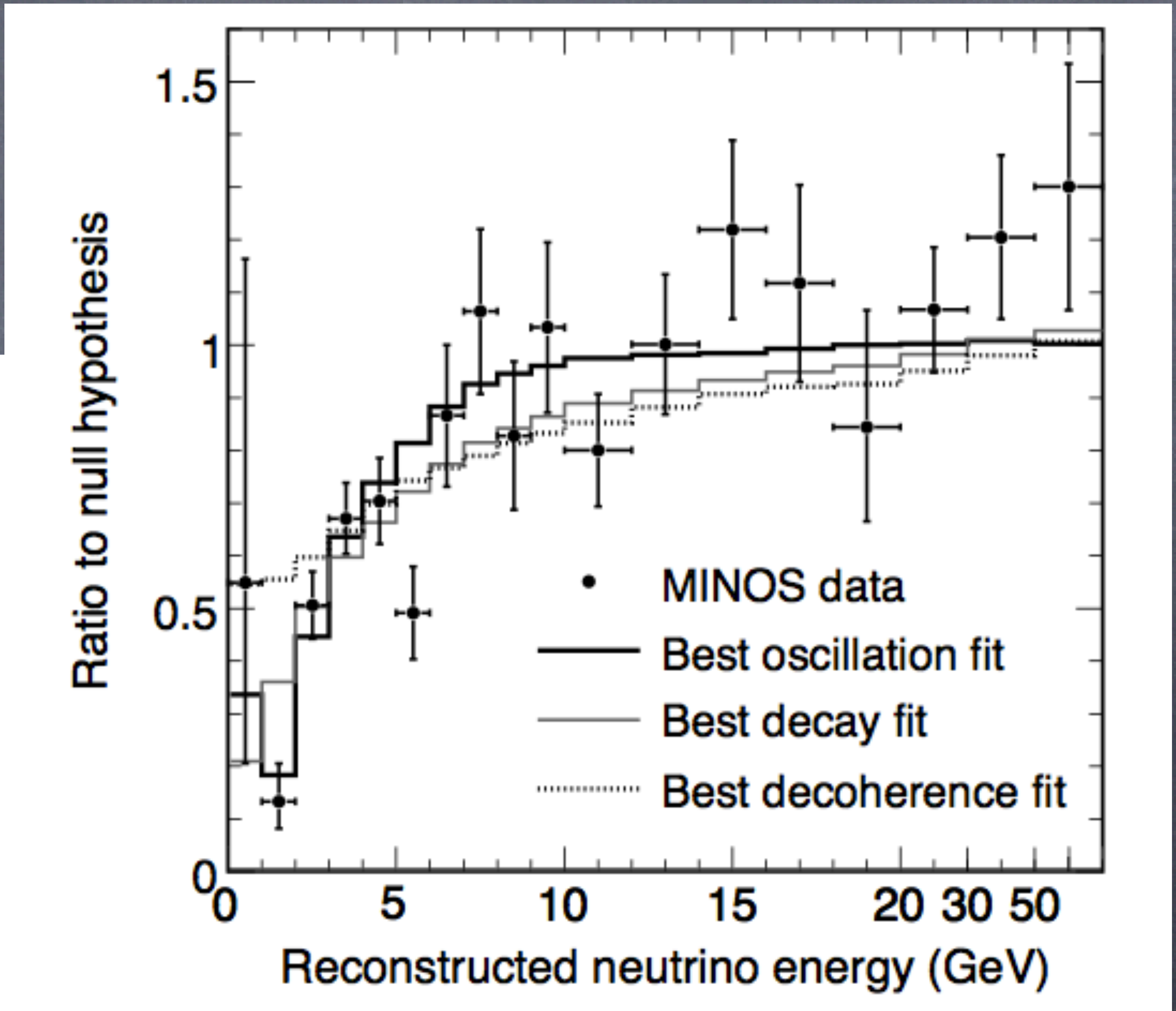
Accelerator Neutrinos

$\pi^+ \to \mu^+ + \nu_\mu$

MINOS :

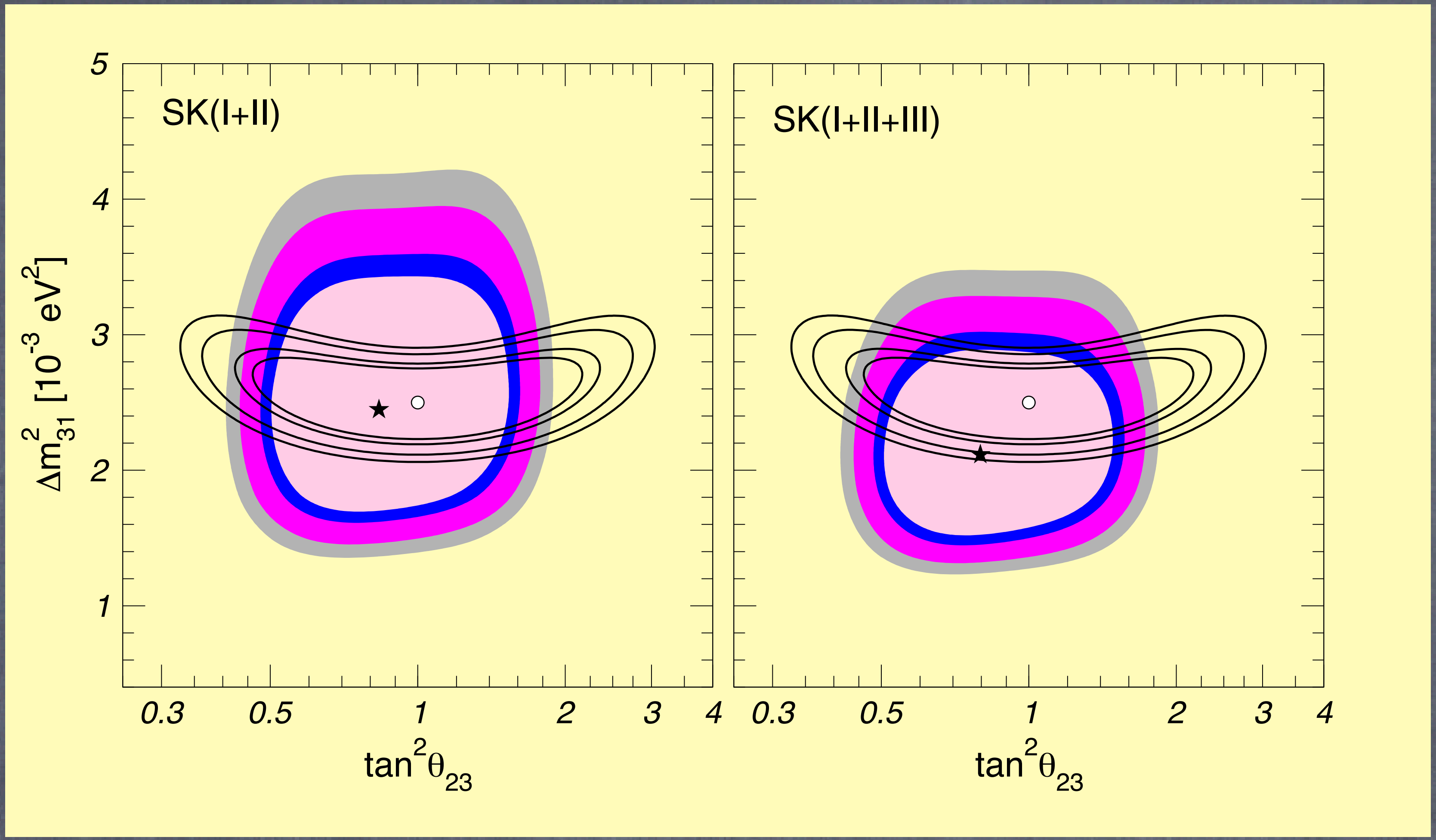
$\nu_\mu \to \nu_\mu$





$$P^{3g}_{\nu_\mu \to \nu_\mu} \sim s^2_{13} \frac{\cos 2\theta_{23}}{c^2_{23}} + (1 - s^2_{13} \frac{\cos 2\theta_{23}}{c^2_{23}}) P^{2g}_{\nu_\mu \to \nu_\mu}(\Delta m^2_{31}, \theta_{23})$$

Fit of Parameters



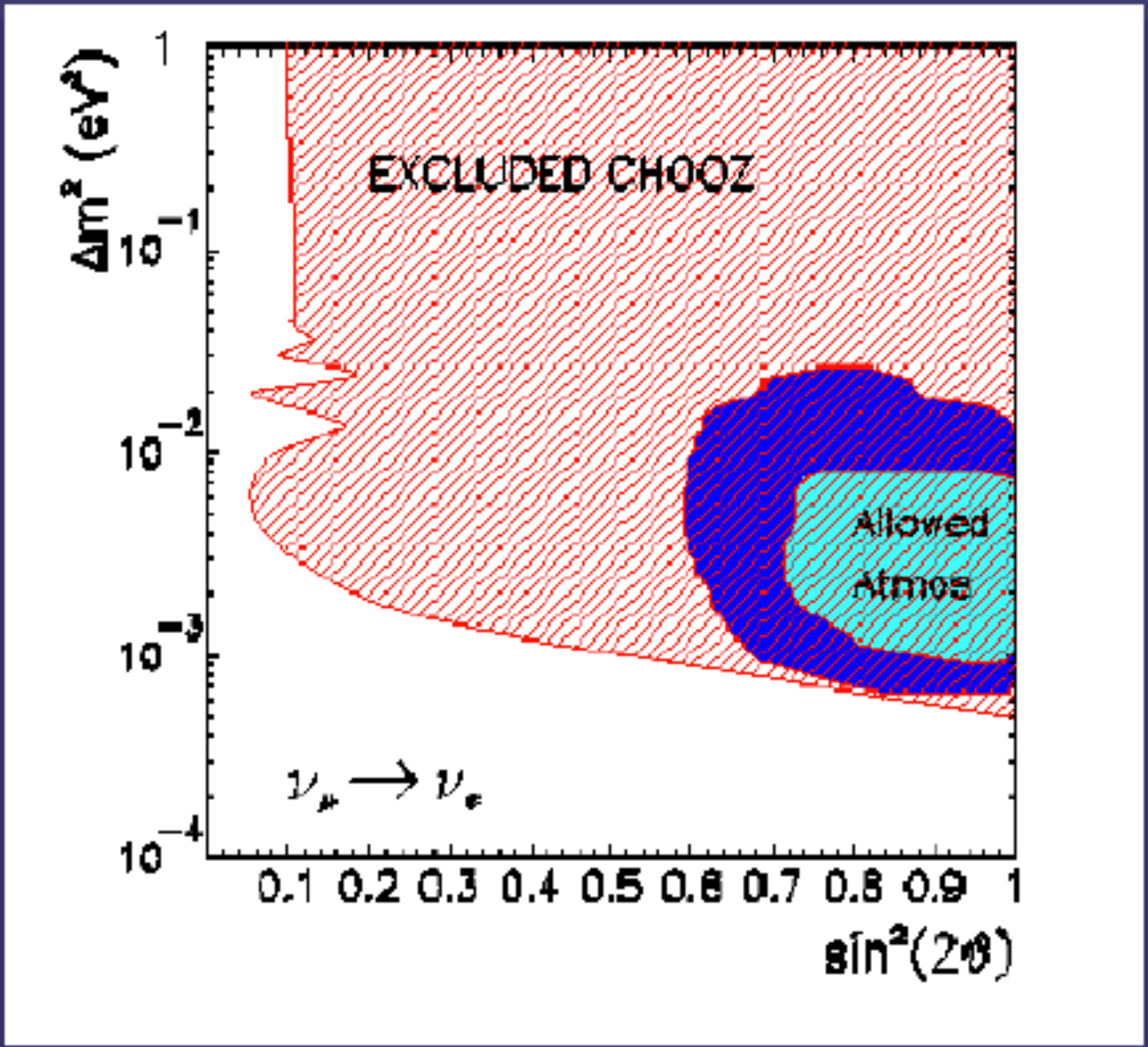
$$\mathbf{\Delta m^2_{31}} = \begin{cases} -2.36 \pm 0.07\,(\pm 0.36) \times 10^{-3}\,\mathrm{eV}^2 \\ +2.47 \pm 0.12\,(\pm 0.37) \times 10^{-3}\,\mathrm{eV}^2 \end{cases} \quad @\ \mathbf{4.3}\%$$

$$\boldsymbol{\theta_{23}} = \mathbf{42.9}\ {}^{+4.1}_{-2.8} \left({}^{+11.1}_{-7.2} \right)^{\circ} \qquad \sin^{\mathbf{2}}\theta_{\mathbf{23}} \quad @\ \mathbf{12}\%$$

[M.C. Gonzalez-Garcia, M. Maltoni, J. Salvado, JHEP 04 (2010) 056]

$$\bar{\nu}_e \rightarrow \bar{\nu}_e$$

CHOOZ (1999)



$P^{osc} < 0,05$

$\sin^2 2\theta_{13} < 0,15$

$\sin^2\theta_{13} < 0,04$

$$L_{ij}^{osc} = \frac{4\pi E}{\Delta m_{ij}^2}$$

$$P_{ee}^{CHOOZ} \approx 1 - \sin^2 2\theta_{13} \sin^2\left(\frac{\pi L}{L_{32}^{osc}}\right)$$

baseline

1 km

$E \sim 3$ MeV

$|\Delta m^2_{31}| \sim 3 \times 10^{-3}$ eV^2

$L^{osc}_{31} \sim 2.5$ km

$\Delta m^2_{21} \sim 8 \times 10^{-5}$ eV^2

$L^{osc}_{21} \sim 100$ km

do not contribute at all

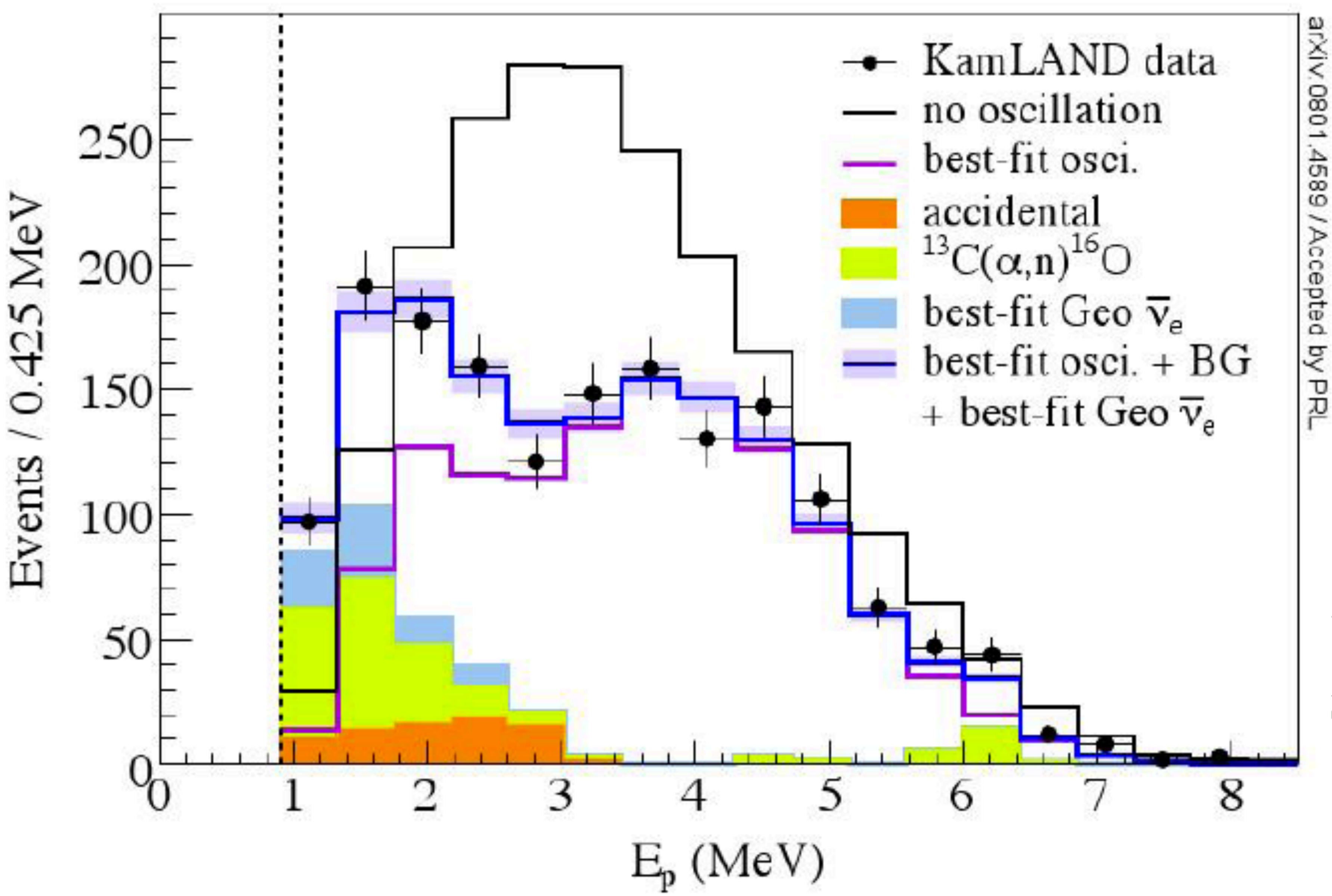
$\bar{\nu}_e \rightarrow \bar{\nu}_e$

KamLAND

$$P^{3g}_{\nu_e \rightarrow \nu_e} = \sin^4\theta_{13} + \cos^4\theta_{13}\, P^{2g}_{\nu_e \rightarrow \nu_e}(\Delta m^2_{12}, \theta_{12})$$

From Mar 9, 2002 to May 12, 2007

1491 live days, 2881 ton-year exposure (3.8x KL2004)



baseline

180 km

$E \sim 3$ MeV

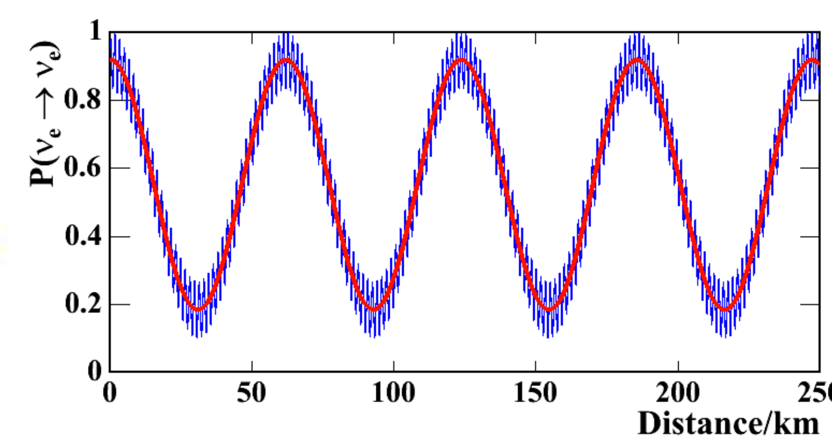
$|\Delta m^2_{31}| \sim 3 \times 10^{-3}\ eV^2$

$L^{osc}_{31} \sim 2.5$ km

averaged !

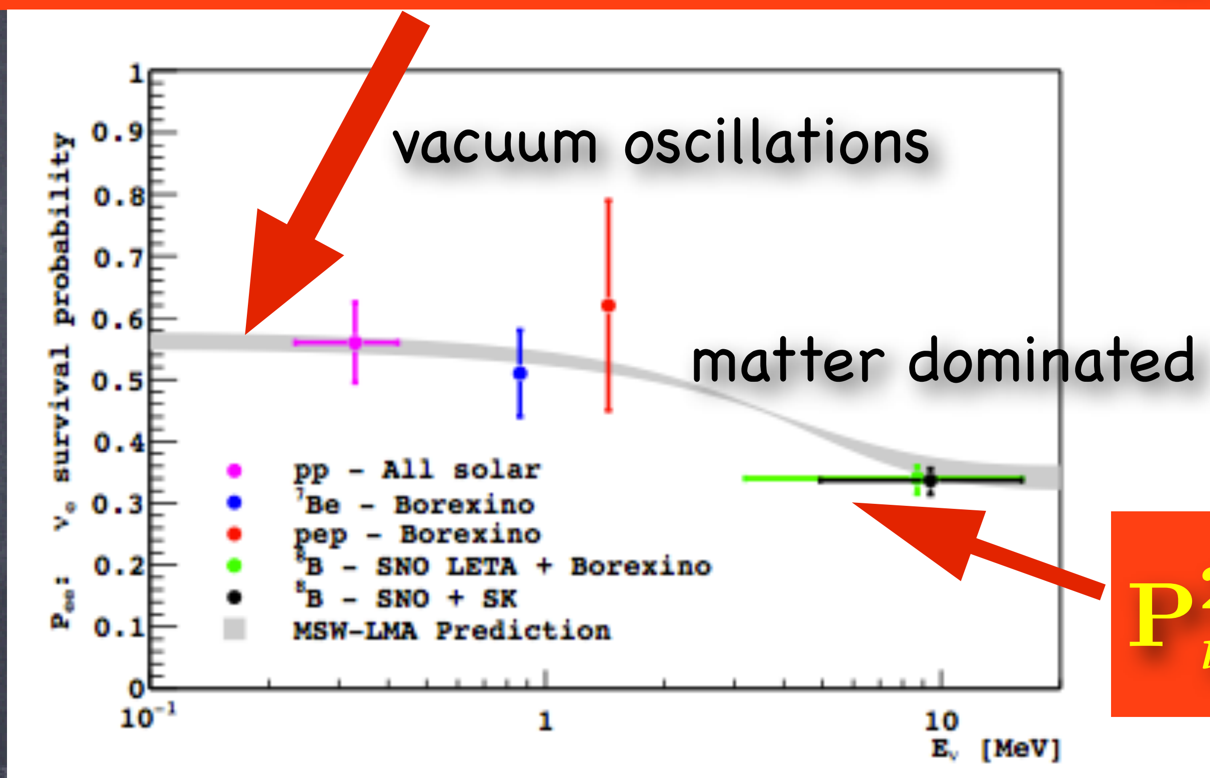
$\Delta m^2_{21} \sim 8 \times 10^{-5}\ eV^2$

$L^{osc}_{21} \sim 100$ km



Solar Neutrinos

$$P^{2g}_{\nu_e\to\nu_e}(\Delta m^2_{12},\theta_{12}) \sim 1 - \frac{1}{2}\sin^2 2\theta_{12}$$



MSW Effect

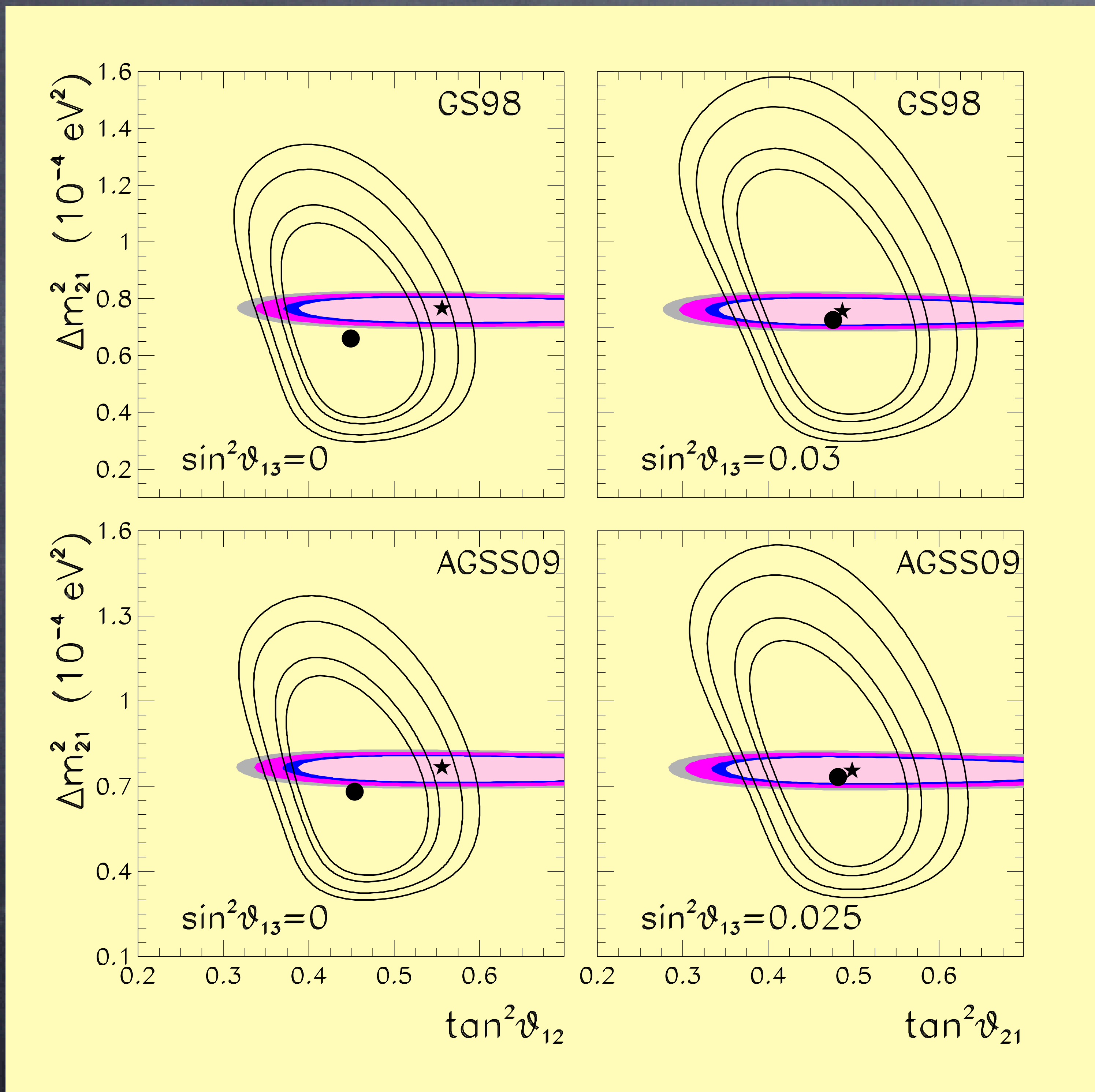
$$P^{2g-mat}_{\nu_e\to\nu_e}(\Delta m^2_{12},\theta_{12}) \sim \sin^2\theta_{12}$$

$$P^{3g}_{\nu_e\to\nu_e} = \sin^4\theta_{13} + \cos^4\theta_{13}\, P^{2g-mat}_{\nu_e\to\nu_e}(\Delta m^2_{12},\theta_{12})$$

$$V_e = \sqrt{2}\, G_F\, n_e \qquad n_e \to n_e \cos^2\theta_{13}$$

$$L^{osc}_{31,32} = \frac{4\pi E}{|\Delta m^2_{31,32}|} << L_{Sun-Earth}$$ *those are averaged*

Fit of Parameters



$$\mathbf{\Delta m^2_{21}} = \mathbf{7.59 \pm 0.20} \begin{pmatrix} +0.61 \\ -0.69 \end{pmatrix} \times \mathbf{10^{-5}}\mathrm{eV}^2$$

@ **2.6%**

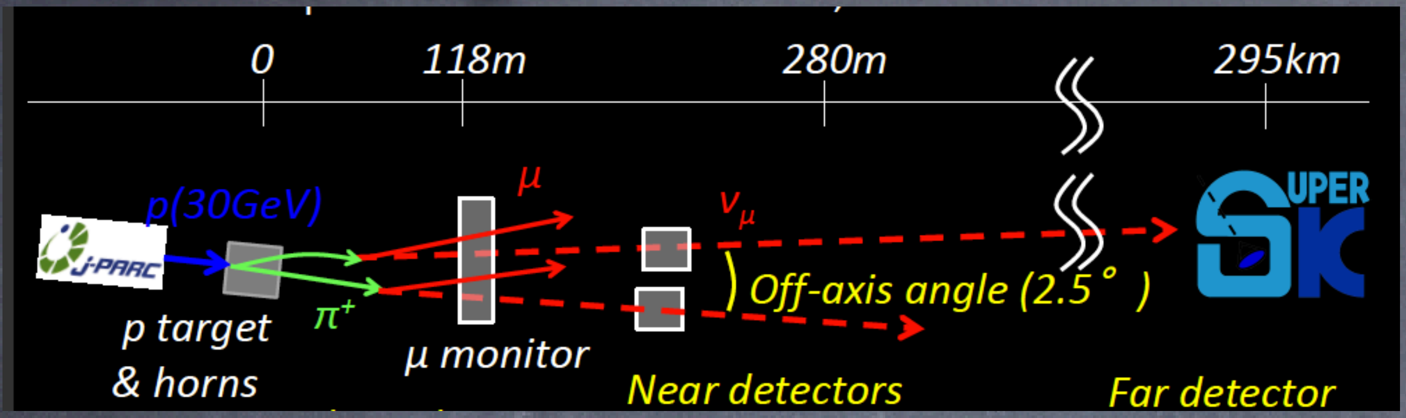
$$\theta_{\mathbf{12}} = \mathbf{34.4 \pm 1.0} \begin{pmatrix} +3.2 \\ -2.9 \end{pmatrix}^\circ$$

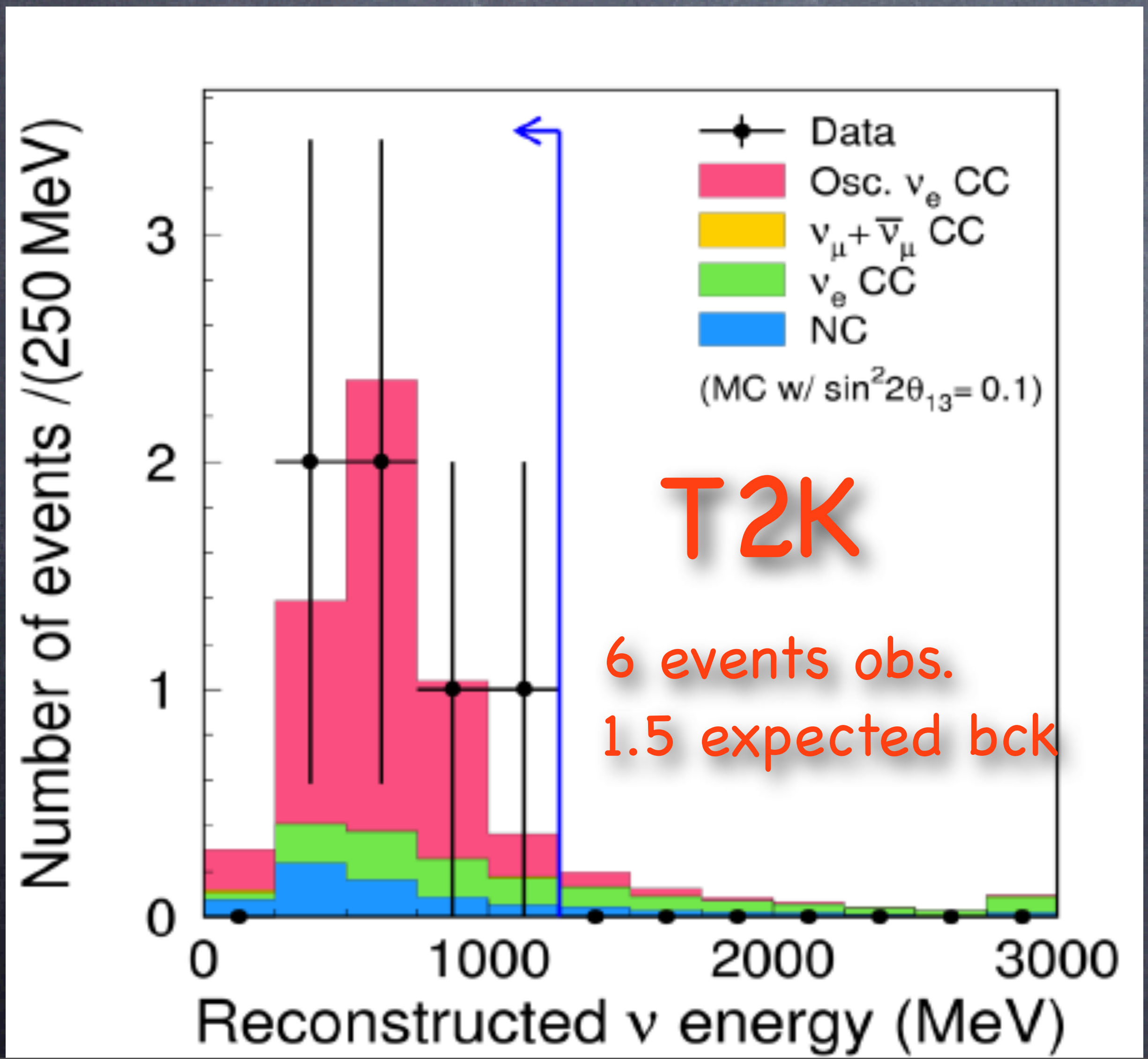
$\sin^{\mathbf{2}} \theta_{\mathbf{12}}$ @ **5.4%**

[M.C. Gonzalez-Garcia, M. Maltoni, J. Salvado, JHEP 04 (2010) 056]

$\nu_\mu \rightarrow \nu_e$

June 2011





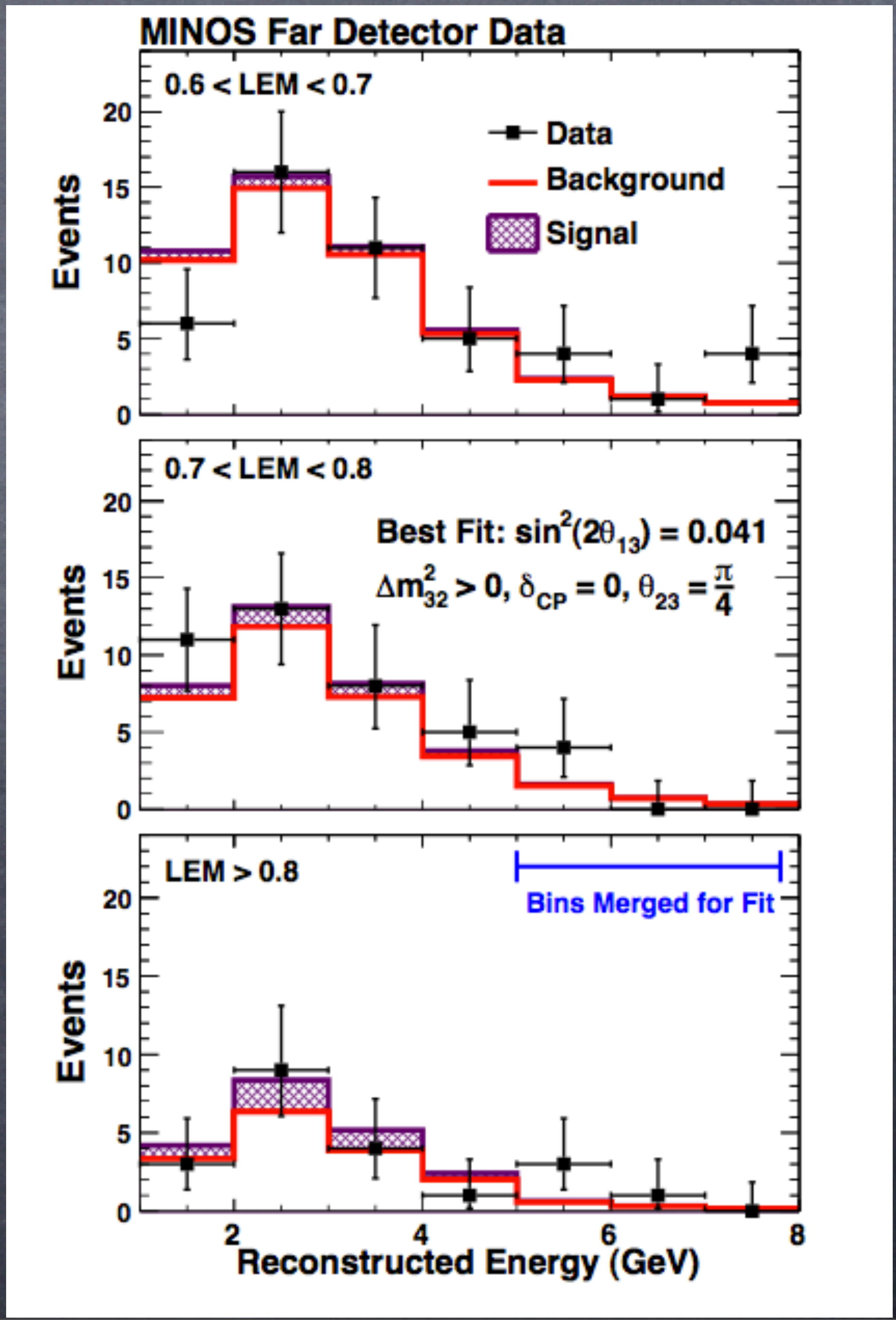
T2K

6 events obs.
1.5 expected bck

K. Abe et al., Phys. Rev. Lett. 107, 041801 (2011)]

MINOS

[P. Adamson et al., Phys. Rev. Lett. 107, 181802 (2011)]

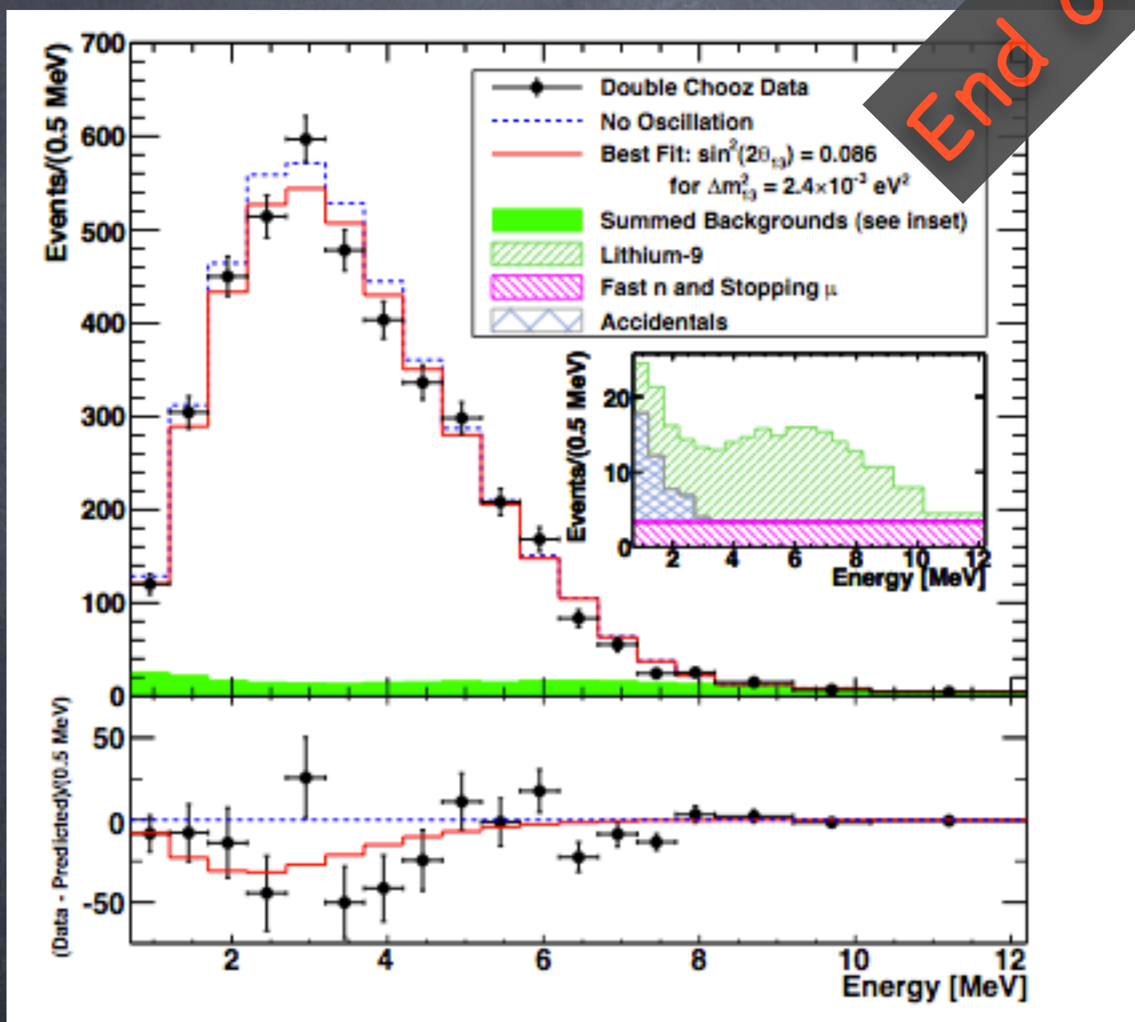


$\bar{\nu}_e \longrightarrow \bar{\nu}_e$

Double Chooz

End of 2011

Y. Abe et al., arXiv:1112.6353

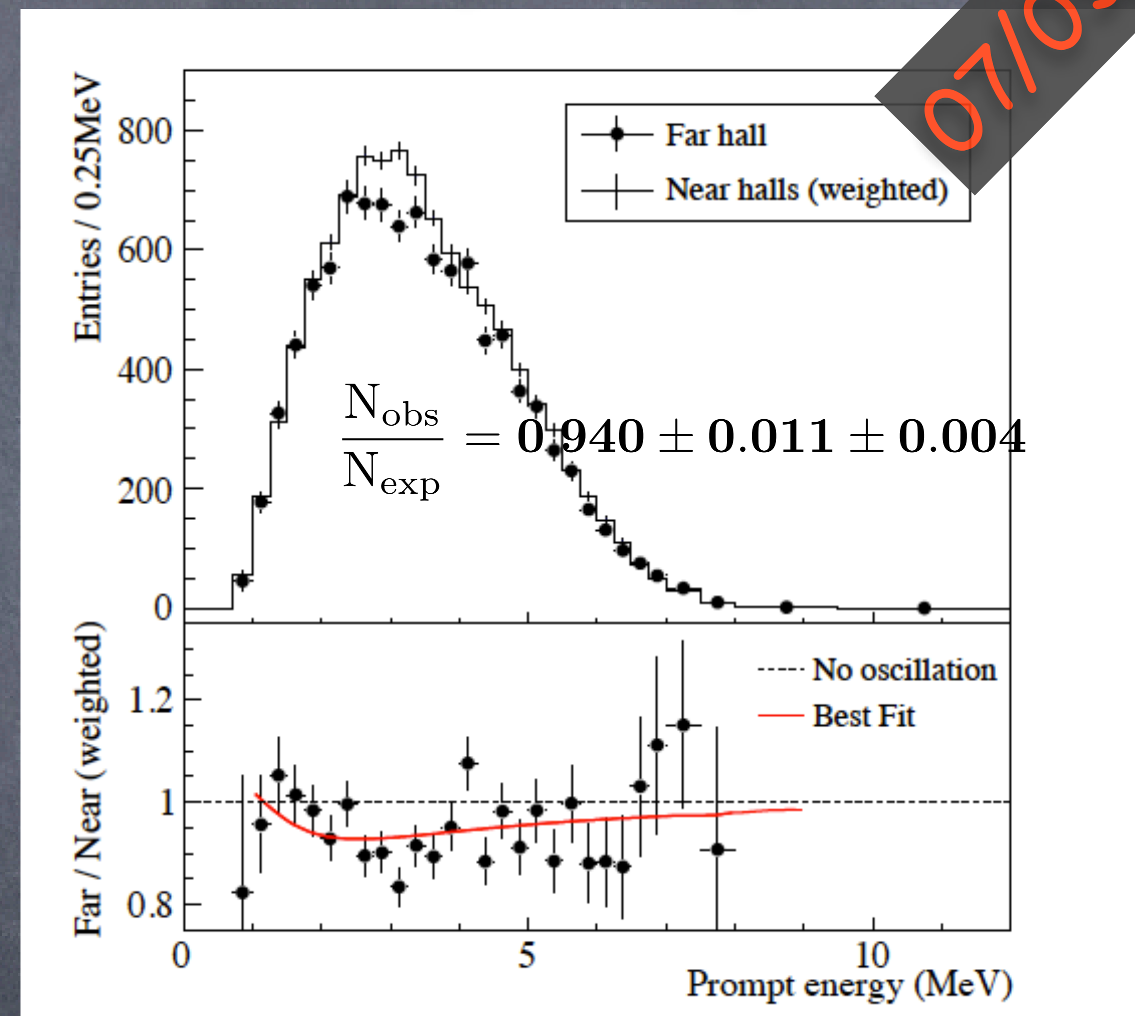


$$\frac{N_{obs}}{N_{exp}} = 0.944 \pm 0.016 \pm 0.040$$

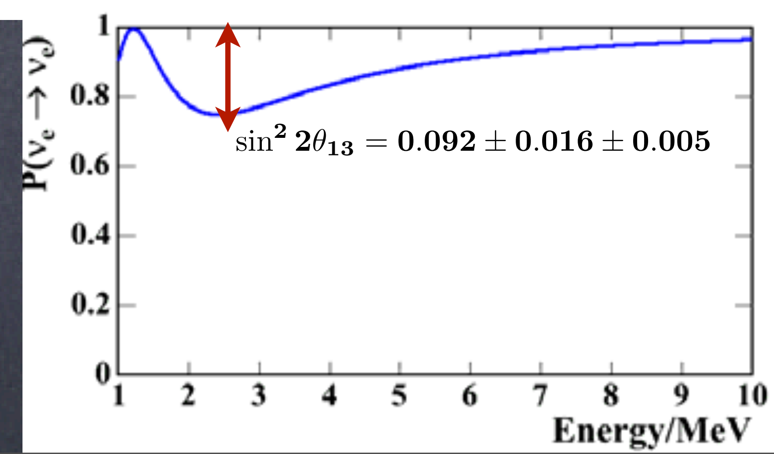
$$\sin^2 2\theta_{13} = 0.086 \pm 0.041 \pm 0.030$$

Daya Bay

07/03/2012



$$\frac{N_{obs}}{N_{exp}} = 0.940 \pm 0.011 \pm 0.004$$



$$\sin^2 2\theta_{13} = 0.092 \pm 0.016 \pm 0.005$$

Fit of Parameters

$$P^{3g}_{\bar{\nu}_e\to\bar{\nu}_e} = 1 - c^4_{13}\sin^2 2\theta_{12}\sin^2\Delta_{21} - s^2_{12}\sin^2 2\theta_{13}\sin^2\Delta_{32} - c^2_{12}\sin^2 2\theta_{13}\sin^2\Delta_{32}$$

$$\mathbf{P^{3g}_{\bar{\nu}_e\to\bar{\nu}_e} \sim 1 - \sin^2 2\theta_{13}\sin^2\left(\frac{\Delta m^2_{32}L}{2E}\right)}$$

$$\mathbf{\Delta_{ij} = \frac{\Delta m^2_{ij}L}{4E}}$$

Reactor Experiments

Fit of Parameters

$$P^{3g}_{\bar{\nu}_e \to \bar{\nu}_e} = 1 - c^4_{13} \sin^2 2\theta_{12} \sin^2 \Delta_{21} - s^2_{12} \sin^2 2\theta_{13} \sin^2 \Delta_{32} - c^2_{12} \sin^2 2\theta_{13} \sin^2 \Delta_{32}$$

$$\mathbf{P^{3g}_{\bar{\nu}_e \to \bar{\nu}_e} \sim 1 - \sin^2 2\theta_{13} \sin^2\left(\frac{\Delta m^2_{32} L}{2E}\right)}$$

$$\mathbf{\Delta_{ij} = \frac{\Delta m^2_{ij} L}{4E}}$$

Reactor Experiments

Accelerator Experiments

sensitivity to δ

$$\mathbf{P^{3g}_{\nu_\mu \to \nu_e} = |2U^*_{\mu 3} U_{e3} \sin \Delta_{31} e^{-i\Delta_{32}} + 2U^*_{\mu 2} U_{e2} \sin \Delta_{21}|^2}$$

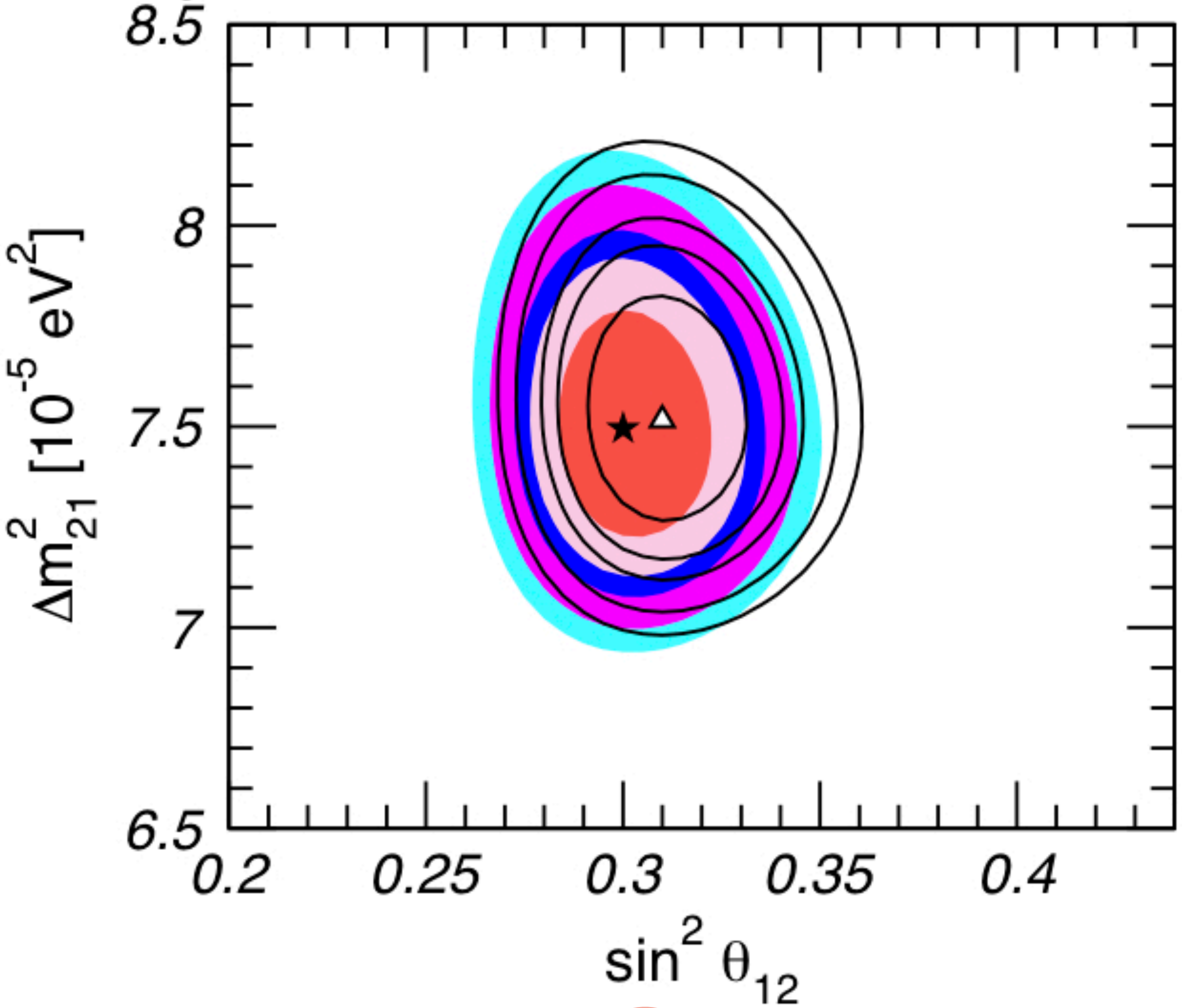
$$\mathbf{P^{3g}_{\nu_\mu \to \nu_e} \sim P_{atm} + 2\sqrt{P_{atm}}\sqrt{P_{sol}} \cos(\Delta_{32} + \delta) + P_{sol}}$$

$$\mathbf{\sqrt{P_{atm}} \equiv s_{23} \sin 2\theta_{13} \sin \Delta_{31} \quad \sqrt{P_{sol}} \equiv c_{23} c_{13} \sin 2\theta_{12} \sin \Delta_{21}}$$

Status of the 3 ν Paradigm

[M.C. Gonzalez-Garcia et al., arXiv:1209.3023]

$$\sin^2\theta_{12} = 0.30 \pm 0.013 \qquad \Delta m^2_{21} = (7.50 \pm 0.185) \times 10^{-5}\text{eV}^2$$



solar neutrinos

+

KamLAND

(4.3%, 2.4%)

$$\mathbf{V} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-\imath\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{\imath\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{\imath\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{\imath\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{\imath\delta} & c_{23}c_{13} \end{pmatrix}$$

Status of the 3 ν Paradigm

[M.C. Gonzalez-Garcia et al., arXiv:1209.3023]

$$\sin^2\theta_{23} = 0.41 \; {}^{+0.037}_{-0.025}$$ **1st octant**

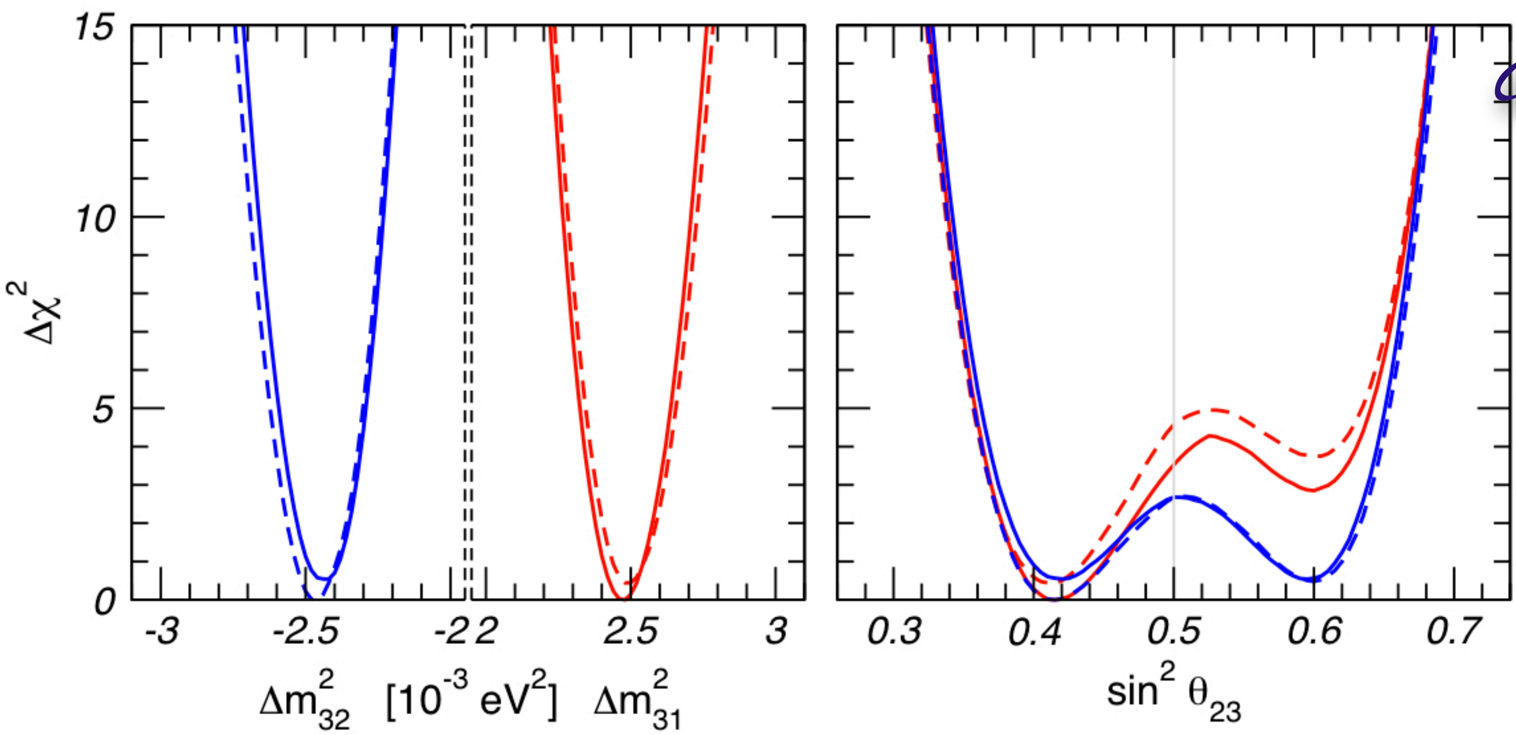
$$\sin^2\theta_{23} = 0.59 \pm 0.022$$ **2nd octant**

Octant Problem

$$\Delta m^2_{31} = (2.47 \pm 0.07) \times 10^{-3}\text{eV}^2$$ **Normal Hierarchy**

$$\Delta m^2_{32} = -(2.43 \; {}^{+0.042}_{-0.065}\;) \times 10^{-3}\text{eV}^2$$ **Inverted Hierarchy**

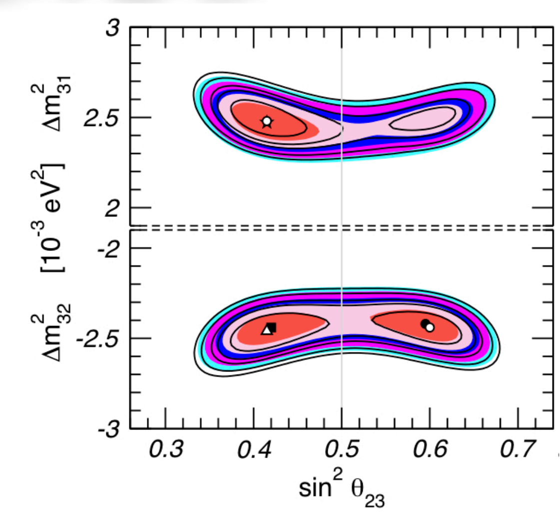
Hierarchy Problem



atmospheric neutrinos

+

MINOS

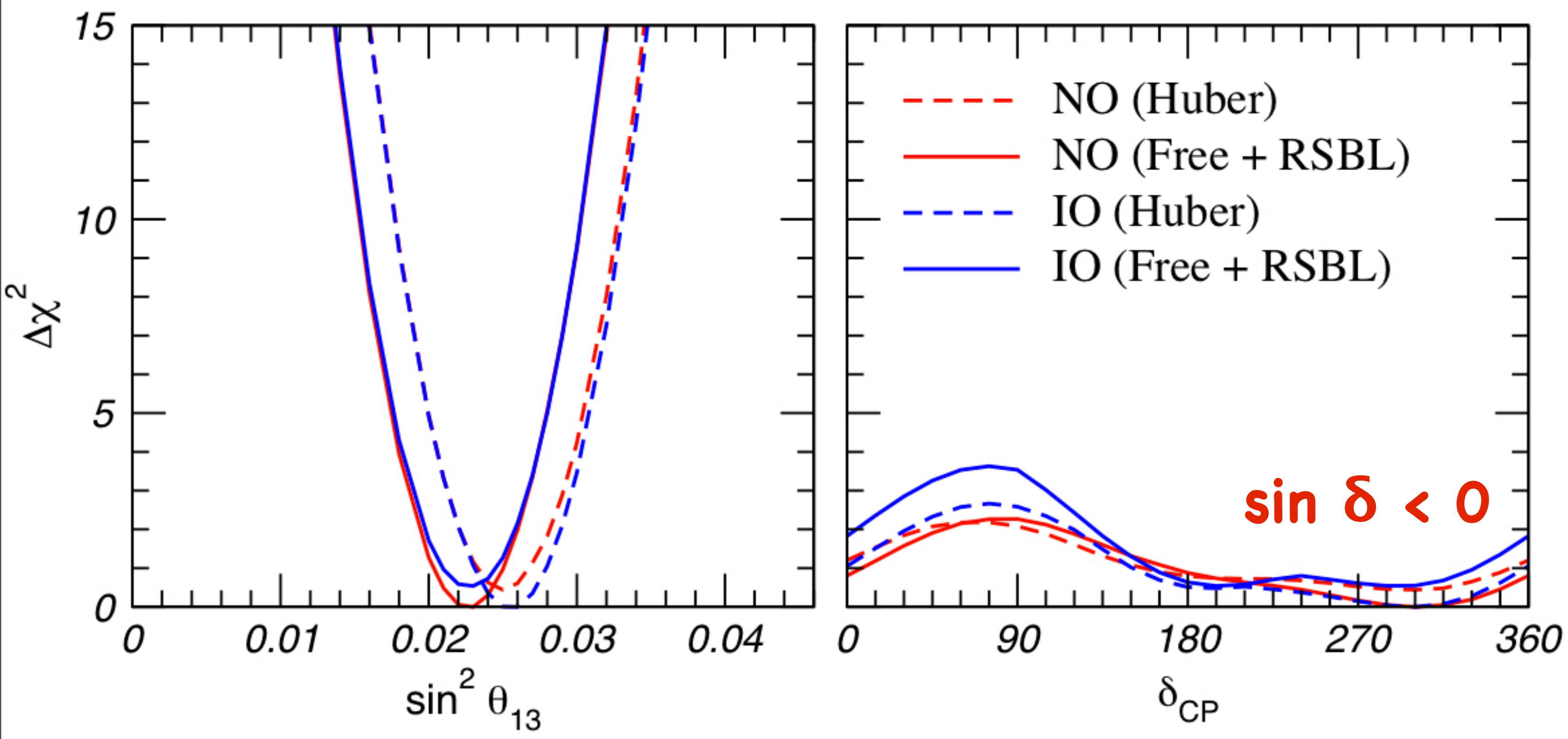


$$\mathbf{V} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

Status of the 3 ν Paradigm

[M.C. Gonzalez-Garcia et al., arXiv:1209.3023]

$$\sin^2\theta_{13} = 0.023 \pm 0.0023 \qquad \delta = (300 \ ^{+66}_{-138}\ )^\circ$$



reactor

+

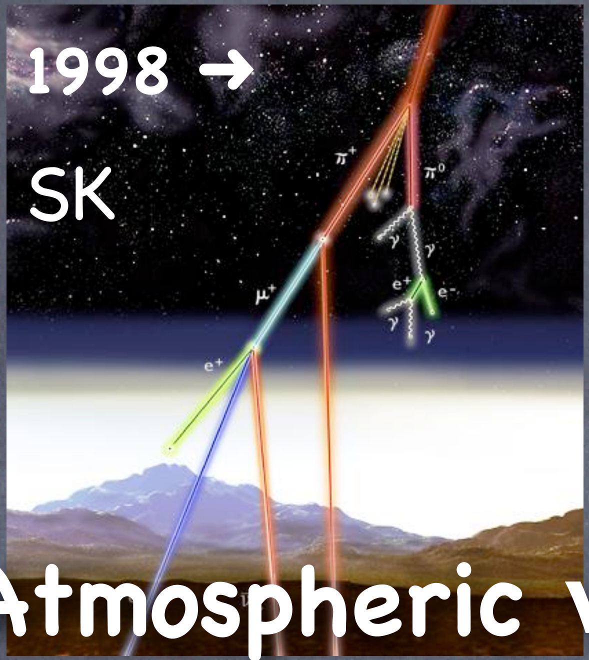
atmospheric

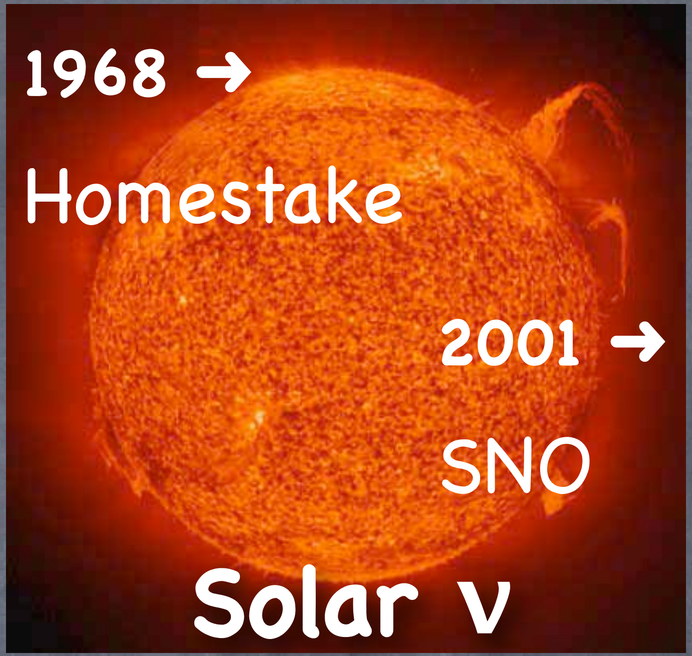
neutrinos

CP Violation?

$$\mathbf{V} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-\imath\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{\imath\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{\imath\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{\imath\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{\imath\delta} & c_{23}c_{13} \end{pmatrix}$$

Nature has Spoken





Neutrino Oscillations ☛ Physics Beyond the Standard Model